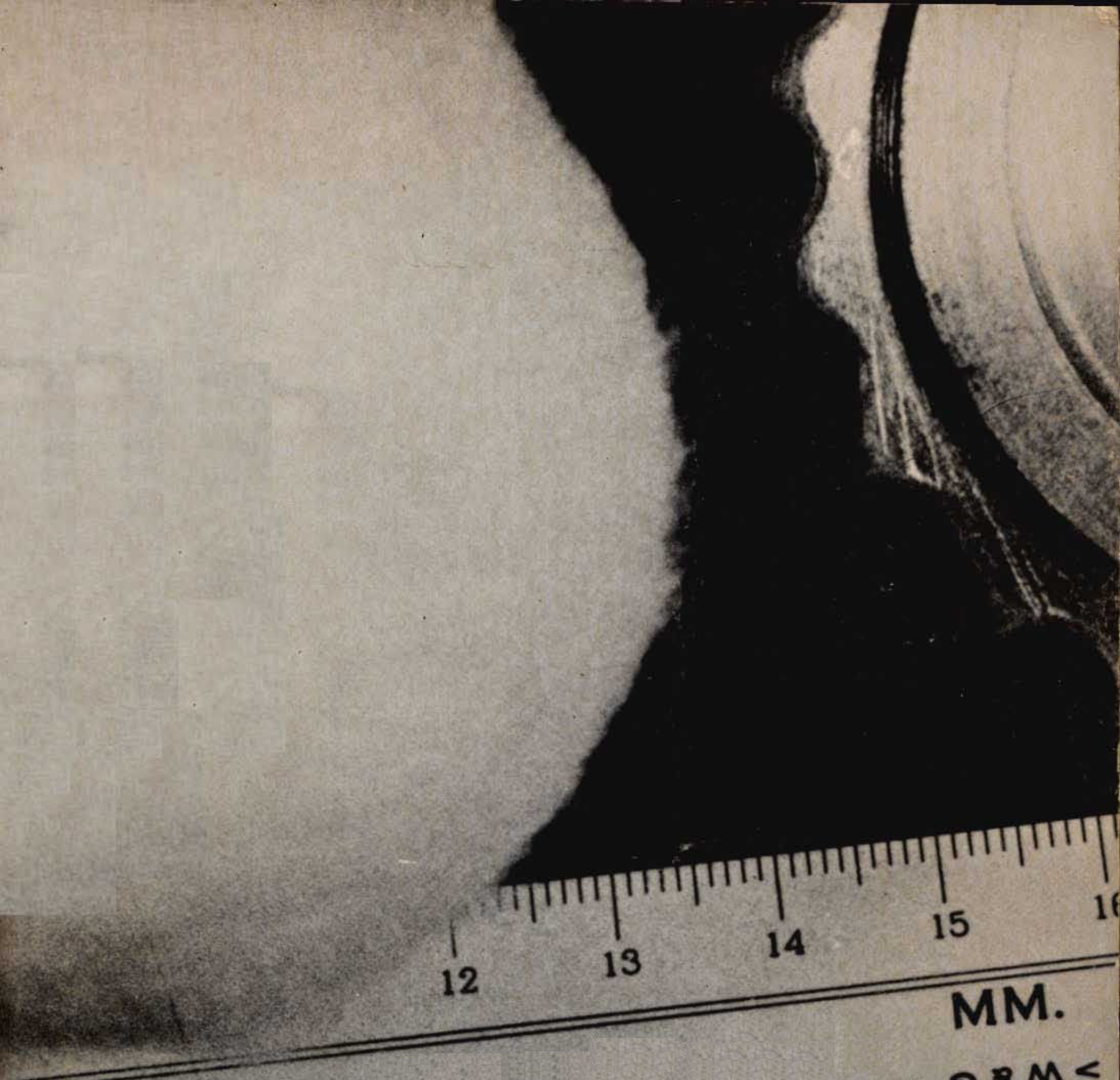

FOSP·S7

THE PHYSICS OF MEASUREMENT

PETER SANDERY

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Diagrams by Ray Black

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INTRODUCTION

This booklet is not intended to tell you all there is to know about measurement in physics—this would be an impossible task. It is intended to make you think about methods for taking measurements and to devise ways of your own of doing this instead of relying upon instructions. It should also lead you to conclude that, although there are acceptable facts in physics, there are no really absolute facts. For this reason the book does not consist of a series of "facts" to be read and learnt.

Many questions are asked throughout the text. The answers may be found by using commonsense, a dictionary, an encyclopaedia, or a suitable physics reference book.

Many of the experiments suggested can be done without the use of expensive equipment—try them. Devise your own experiment where possible, but always keep in mind the conditions under which valid, reliable measurements may be made during the experiment.

When you have completed the text you should have gained a little "factual" knowledge, but more important, you should have also gained a fresh, active approach to the methods and reasons for measurement in physics.

Professor Richard Feynman, Professor of Theoretical Physics at the Californian Institute of Technology, recently said: "It is not science to know how to change Centigrade to Fahrenheit. It's necessary, but it is not exactly science . . . Science is the belief in the ignorance of the experts".

In order that you may question the experts, you must have a spirit of enquiry—an inner drive to *know*. If you have this quality you will appreciate its value.

Fig 0.1 Question: how long was the exposure? Note: a protractor is a transducer (360° = 24 hours), and page 5 and Chapter 4 will tell you what a transducer is. What is the instrument in the foreground?

1: MEASUREMENT—Why is it important?

You have all probably made thousands of measurements in your lives, but have you ever stopped to consider what it involves? What do you actually do when you make a measurement?

Whether it is the length of a piece of wood, the mass of an object or the period of time between two events, you are actually making a *comparison* against some familiar *standard*.

The earliest and simplest standards were probably the personal ones—handspans and paces for length, pulse beats for time and so on. However, at a very early stage in the history of civilisation, the first objective standard weights and measures were introduced. The comparison then became indirect. Instead of comparing directly with the standard, comparison was made with a *copy* of the standard, more or less accurate. Inspectors of weights and measures were among the first public servants.

Today, many measurements are made even more indirectly via some form of *transducer*—a device to convert one quantity into another which is more readily measurable, as a thermometer converts *temperature* into *length*.

All measurements can be reduced to comparison with some standard.

To test this idea, try the following. If you wear ear plugs and a blind fold, what measurements can you make? Do this as a "thought" experiment. You should be able to think of some.

Before discussing some of these ideas in detail perhaps we should consider why it is important:

(a) to make measurements, and

(b) to know how reliable these experiments are

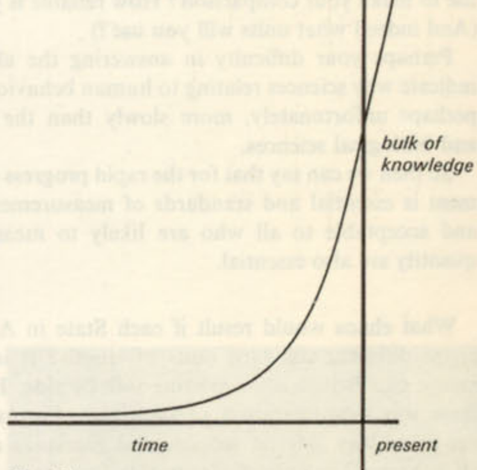


Fig 1.1

If one looks at a graph of scientific knowledge plotted against time, it looks something like Fig 1.1.

The point at which the bulk of scientific "fact" started to rise steeply coincides very well with the introduction of experimental (as opposed to philosophical) methods and to the use of measuring instruments and accepted standards.

Behavioural sciences in which no direct measurement and comparison with agreed standards is possible, have progressed very slowly. It would also seem that the advancement of a particular science depends on the sophistication of its associated measuring devices. We cannot always rely on our own senses to make measurements. Firstly, we can be mistaken—optical illusions are only a dramatic illustration of the shortcomings of our eyesight. Secondly, the range of our senses is restricted—we can, for example, only 'see' a limited part of the electromagnetic spectrum, and have to use transducers for the rest.

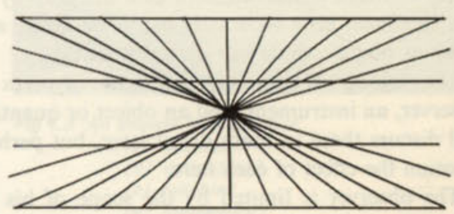


Fig 1.2 Are any of these lines curved?

How well do you think you could estimate the length of a self-luminous white rod in a completely darkened room, if you did not know how far away the rod was, i.e. if you had no familiar object or background to refer to?

Now do Experiment S7/1, page 59

We need measuring instruments and preferably reliable, valid instruments if we are to progress in Physics.

These days, closely allied with scientific advance there is a growing public feeling that science breeds some "ugly" products—nerve gas, bacterial warfare, nuclear bombs, pollution of various sorts. This seems to indicate that a scientist has a moral obligation to the community to see that his work aids his society and does not seek to ultimately destroy it. Perhaps there should be a scale of moral qualifications as well as academic qualifications, required for positions in which scientists are likely to become involved in research leading to potentially destructive products. If so, how do you measure moral quality? What is your standard? What do you use to make your comparison? How reliable is your measurement? (And indeed what units will you use?)

Perhaps your difficulty in answering the above questions will indicate why sciences relating to human behaviour have progressed, perhaps unfortunately, more slowly than the physical, chemical and biological sciences.

So then we can say that for the rapid progress of science, measurement is essential and standards of measurements that are known and acceptable to all who are likely to measure any particular quantity are also essential.

What chaos would result if each State in Australia decided to adopt different standard units of length? It is bad enough with metric and British units existing side by side. How much worse if there was a proliferation of standards. Fortunately, the trend is the other way. Metric weights and measures are rapidly ousting all others. For scientific purposes, we need even more precise standards for any quantity that is to be subject to measurement. We shall consider just exactly what these standards should be at a later point.

Now it is perhaps important to consider the second point raised earlier. It is all very well to have a standard that is used by all, but when we make a measurement of a quantity by comparing it with this standard, how accurately can it be done, and is it important to know how accurate our measurement is?

The taking of any measurement involves three things—an observer, an instrument, and an object or quantity to measure. We will discuss these in more detail later, but perhaps we can briefly mention the effect of each here.

The observer is limited by the scope of his senses, e.g. events that occur in time intervals of less than one-twentieth of a second lose their individuality and merge into one another because of retention of vision of the eye. He is also limited by his skill in using a measuring instrument—a sensitive balance in the hands of an inexperienced user is hardly likely to result in an accurate measurement of a mass.

The instrument selected for a particular measurement should have a range suitable for the job and all instruments used in measuring some derived quantity should yield results with the same degree of accuracy. We shall assume that the observer is sensible enough to check the instrument for zero error, faulty functioning, etc.

The quantity being measured or estimated also limits the accuracy of the result. Imagine trying to measure the size of an amoeba (what is an amoeba?) underneath a microscope provided with a scale in the field of vision. How accurate would your result be? It should not be difficult to see that a poor observer, faulty instrument and ill-defined quantity can achieve a result that is of very doubtful validity.

Let us try another "thought" experiment. Suppose you were given a school ruler that looked much the same as any other, but was in fact only 11.9 inches long, divided up into 12 divisions with the normal inch markings on it.

Would you be able to tell immediately that it was not 12 inches (12 standard inches, that is) long? How?

Right—now suppose that everybody in the class had such a ruler and no other measuring instrument, could you tell if anything was unusual about the ruler? I doubt it!

If you were unaware that the ruler was non-standard all of your measurements would have one source of error right from the start (regardless of any others you might encounter).

If you were aware of the error, you could make allowance for it without too much trouble.

How many times have you looked at a clock, a speedometer, a meter of any sort and taken the reading given as exact. In fact, what does *exact* mean anyway? Many things apart from the faulty meter can influence the accuracy of the device you are observing—the zero (is it zero?), the thickness of the needle and the graduations, *parallax* (a rather complex word—what does it mean?), the conditions at the time of measurement.

You should always question the reliability and accuracy of any measuring device that you use. If it is inaccurate (and they all are to *some* degree), find out the degree of inaccuracy and, if it is acceptable, allow for it—if not, find a more suitable measuring instrument.

We have considered why we should measure and why we should know the accuracy of the measuring instrument. Now we should discuss exactly what we are likely to want to measure and what units we want to use to do this.

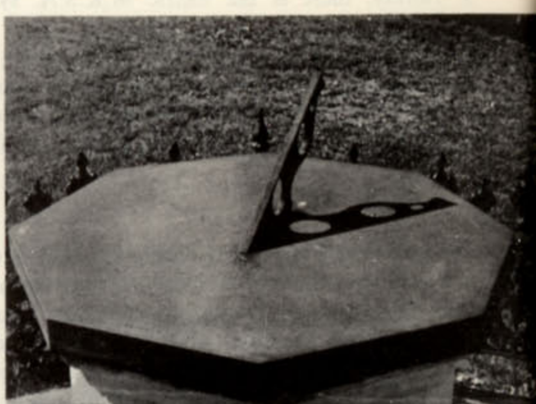


Fig 1.3 An ancient form of clock.
Is it (a) accurate?
(b) consistent?
(c) reliable?
(d) convenient?
What are its good points?

2: MEASURABLE QUANTITIES

The fundamental quantities that we may wish to measure are those of length, mass, time and electrical current.

1. The British or F.P.S. system of units was once used widely in all measurements. Nowadays, F.P.S. units are rarely used in scientific work and are gradually being replaced in everyday use. The initials stand for the fundamental units of length, mass and time—*foot, pound* and *second*.

LENGTH	F	FOOT
MASS	P	POUND
TIME	S	SECOND

If you refer to a good dictionary for a definition of the "foot", you will find that it was initially taken as the length of an adult man's foot. Measure your left and right feet. Are they the same length? Are either of them 12″ long or one (standard) "foot" long? Make a survey of the length of feet in your class. Find the average length and the maximum deviation either way. Is the average foot length equal to one "foot"? Can you suggest a reason for any discrepancy?

Make a survey of all the British Units of length that you can find. Probe into history a little and see how they were originally defined. Comment on the validity of these definitions and discuss why most of the units are no longer in common use.

2. The second system of units widely used today is a metric (or power-of-ten type) system—the C.G.S. system. The initials again stand for the units—*centimetre, gram* and *second*.

LENGTH	C	CENTIMETRE
MASS	G	GRAM
TIME	S	SECOND

3. Thirdly, there is the metric M.K.S.A. system of units, the initials standing for *metre, kilogram, second* and *ampere*, now widely used in scientific work and gaining in popularity in common use. Most international athletic events use the metre, as does much international trade and commerce. We will therefore examine the M.K.S.A. system in greater detail.

LENGTH	M	METRE
MASS	K	KILOGRAM
TIME	S	SECOND
CURRENT	A	AMPERE

1. LENGTH

The metre came into being during the days of the French Revolution, and was originally designed to be *one ten-millionth the distance from pole to equator* as ascertained by the actual measurement of an arc of the meridian. This was calculated, and "standard" lengths one metre long were soon in common use. It was then discovered that the measurement of the pole-equator distance was incorrect. Nonetheless, the length that had previously been accepted as the metre was retained.

LENGTH	M	METRE
MASS	K	KILOGRAM
TIME	S	SECOND
CURRENT	A	AMPERE

The metre is now defined in terms of the wavelength of the element krypton. The metre is the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom. Why is this a more satisfactory standard than the earlier one?

Learn the prefixes that are used in the M.K.S.A. system to represent larger and smaller lengths, e.g. one kilometre is 1000 metres, one millimetre is one-thousandth of a metre. (See p. 49)

Fig 2.1 An aerial photograph of Adelaide. Work out a suitable scale using known objects (ovals, etc.) as reference objects. You could also do this by consulting a street directory for lengths of streets. What is the use of such a photo? What information does it give you?

Fig 2.2 The radiophysics division of Australia's Commonwealth Scientific and Industrial Research Organisation has produced this radio telescope designed specifically to pinpoint and photograph small areas of the sun. It is able to focus on less than one per cent. of the sun, and show 10 000 times more detail than a first-class radar set. The telescope was invented by Dr. Wilbur Christiansen, of the radiophysics division of C.S.I.R.O., Sydney, and was set up in a site about 30 miles from Sydney.

What area would 1% of the visible face of the sun encompass? What difficulties would one be likely to encounter in trying to focus like this to obtain fine detail?

2. MASS

LENGTH	M	METRE
MASS	K	KILOGRAM
TIME	S	SECOND
CURRENT	A	AMPERE

The fundamental M.K.S.A. unit of mass is the kilogram, and this is the mass of a standard lump of material kept in the International Bureau of Weights and Measures in France. All other one kilogram masses should be exact copies of this. Do you think they are? (Be careful before answering this to consider what we mean by *exact*; is it possible to have an "exact" copy of anything?) Examine a laboratory standard one kilogram mass. Get some idea of how much brass, wood, water, etc., constitutes one kilogram.

Having established the unit we are going to use to measure mass it may now seem sensible to ask ourselves "Just what is 'mass'? What are we really measuring?"

Well, most of you could quickly reply that mass is the amount of matter in a body and this might seem to be a perfectly adequate answer—adequate that is, until someone asks you what 'matter' is!

Our idea of mass is confused by the fact that it seems to play a dual role. We are aware of our own mass because of the gravitational effect the mass of the earth has on it. We know this effect exists and we rely upon it every day in many ways. Think of how difficult it would be to walk if our muscles had to lift our legs for each step and also make them move downwards.

This property that objects have that we call mass somehow causes an attractive force between two or more objects (always attractive as far as we know). This force is quite weak compared to the other forces in nature, but it acts over large distances. It is the 'glue' that holds the solar system and the Universe together. The attractive force between two masses is readily observable, but why it should exist, and how the force is transferred (how does one mass 'know' of the existence of the other?)—these questions are not so easy to answer. If you are interested in following up this question, see what you can find out about the word *gravitation*.

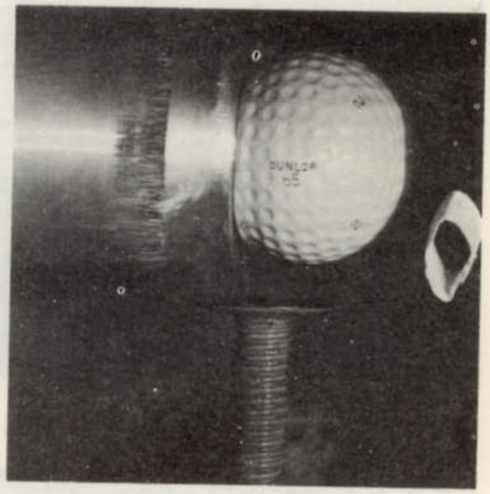


Fig 2.3(a) What does this photo illustrate? What energy changes have taken place? What energy changes are about to take place?

We can compare masses then by comparing the effect that the earth's gravitational field has upon each. This works so long as mass is directly proportional to the weight, which appears to be the case.

Mass also seems to play another role—it determines how hard it is to change the state of motion of a body. Imagine that you run flat out at a cube of polystyrene foam of side six feet in an effort to shift it. Now imagine the same act when the foam is replaced by an equal volume of concrete (solid, well-set, reinforced concrete). The concrete would resist your trying to change its state of motion rather more than the equal volume, but less massive block of foam.

(b) Discuss the incident in this photo
(1) in physical terms
(2) in terms of road safety
Look at the rear seat passenger in the front car—why is he in the position shown?

Mass then can also be a measure of a body's sluggishness or *inertia* — that property of a body to remain at rest or to continue moving in a straight line with uniform velocity unless acted on by an unbalanced force.

On earth you could not move 216 cubic feet of concrete concentrated in one solid block, but imagine that you were in orbit around the earth and so was the block. In such a situation you would find that you could move the block if you applied a small constant force (this could prove difficult—think why), but you would have to be satisfied with moving it slowly—initially at least,

Having succeeded in moving the block relative to yourself, with difficulty, you would find it equally difficult to cancel out the motion of the block by pushing on it in the opposite direction. You could bring the block to rest, but you would have to be prepared to do it slowly.

When we measure mass then, we are measuring that property of a body that determines the interaction of it with another mass, i.e. its "weight-like" property (called the gravitational mass), or its inertial property (inertial mass). We have not really said what mass is, we have merely discussed the behaviour of a body which has this property we call mass.

Albert Einstein put forward a now famous equation relating mass and energy.

E (energy) = m (mass) × c^2 (velocity of light, squared)

Now do Experiment S7/2, page 60

So mass and inertia, mass and 'weight', mass and energy are all related, and when we compare masses by some form of measurement we could equally well be comparing energies.

Balances — Weighing errors

Gravitational mass may be measured on a standard beam balance, triple arm balance, electronic balance, etc.

We normally refer to this process as *weighing* and there are several sources of error in the process. Some of these can be attributed to the particular balance, but some depend on the interaction of the mass being 'weighed' with its surroundings. For example:

(1) The substance may absorb water vapour or impurities from the air, or alternatively it may dry out.

(2) There may be apparent changes in weight because of buoyancy effects (the upthrust of the air), electrostatic forces, etc.

As the mass determination becomes more precise, then more attention must be paid to these sources of error.

3. TIME

The fundamental M.K.S.A. unit of time is the second. Many ways have been devised to define an interval of one second. It has been defined as a certain fraction of the year 1900 (is one second now the same interval of time as a second was in the year 1900 or in the year 2000B.C. for that matter? Could you devise some means of checking your answer?)

LENGTH	M	METRE
MASS	K	KILOGRAM
TIME	S	SECOND
CURRENT	A	AMPERE

Calculate the order of magnitude (the power of ten) of one year in seconds.

The second is now defined using the atom caesium. Caesium atoms absorb and emit radiation whose frequency is 9 192 631 770 vibrations per second. These atoms form the basis of the *atomic clocks* used as time standards throughout the world.

Looking at the figures quoted above, you might think that this is a remarkably accurate time standard, but think a while. The phrase *vibrations per second* is used—which second is this? The caesium clock has been compared with some previously defined standard. Neither of these are really natural units of time; they are arbitrary man-made units of time.

As with mass, having defined our unit of time we should ask ourselves "What is this stuff we measure in seconds?" Some students answers to the question "What is time?" are shown below:—

(1) "Time is infinite, or is infinity".

(2) "Something verbally indefinable, but which has a very great effect on our lives".

(3) "A sequence of events".

(4) "Time is due to the motion of our planet, Earth, around the sun, i.e. day and night".

(5) "The measure of a happening in relation to anything".

(6) "A fourth dimension".

(7) "Something I haven't got enough of".

Albert Einstein, when asked the above question replied, "Time is what you measure with a clock" and this is as good a definition as any. Time, like any other concept in nature, is really dependent on the laws in which it is employed. All that we ask of our time measuring instruments is that they be consistent.

Of the definitions of time given above, (3) is perhaps worthy of further mention. We unconsciously measure time by placing events in a specific order. Changing events are described by the passage of time (or is the passage of time described by the changing events?)

You may like to ponder the following questions:—

Does time have an absolute zero? i.e. a beginning? (and hence perhaps an end, according to the prophets of doom). If so to what can you relate either end of this then established time scale?

Can man interfere with the passage of time? (i.e. make it go faster or slower?)—this will be discussed later in a little more detail.

Is it possible to go back in time? Suggest reasons.

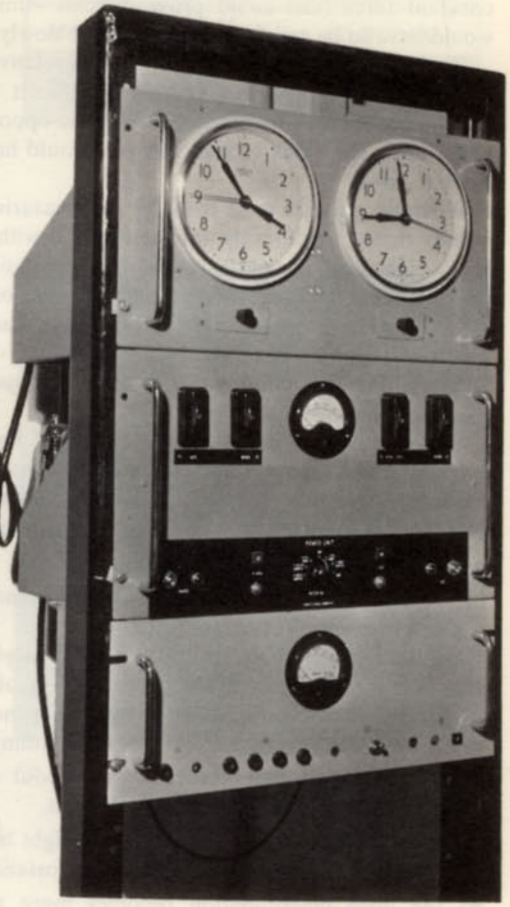


Fig 2.4 Twin clocks in the Mullard Radio Astronomy Observatory, Cambridge, show G.M.T. and Sidereal Time.

Try looking through reference books in the library and make a summary of the different types of clocks that man has devised to help him "tell the time". Try to compare them. See how the relative merits of each (and their limitations) are related to the prevailing conditions of the historical 'time' in which they were used.

In some of the realms of science, it has not always been possible to perform the measurements required to test a scientific theory at the time the theory was put forward. Today it is possible to *simulate* the conditions required for an experiment using a modern electronic computer. The experiment itself need not be done in some cases, the simulated experiment suffices. We do not propose to try to construct such a simulation at this stage, but as a first step towards this process (which you will almost certainly be involved in if you pursue a scientific career), we shall present an example (Exercise S7/3) of how a computer can replace pencil and paper to perform a calculation and to predict what measurements we might be required to make in an actual experiment. This is a valid procedure and although in the simple case we have chosen it may not seem important, in more complex cases the results of the computer calculations do indicate the order of magnitudes of the results of an experiment, and hence the measuring technique required.

Now do Experiment S7/3, page 61

4. ELECTRIC CURRENT

The unit of electric current in the M.K.S.A. system is the ampere. This is defined in terms of the force it produces between two parallel conductors. The definition will be given in the core material you study. Try to discover how the unit was previously defined. What do you think is the maximum electric current that may flow through the wiring of your house without causing damage? (A look at the fuse box will quickly supply you with an estimate.)

LENGTH	M	METRE
MASS	K	KILOGRAM
TIME	S	SECOND
CURRENT	A	AMPERE

THE VERY LARGE AND THE VERY SMALL

We have discussed the units that we are going to use in our measurement of the fundamental quantities of Length, Mass, Time and Electric Current. We have even tried to discuss just what these fundamental quantities represent.

Normally we are quite familiar with measurements of quantities that we can relate to ourselves, to our own height, size, mass and lifetime. We find little difficulty in closing our eyes and visualising familiar quantities, but it is difficult to visualise a mass of 10^{55}kg, a time of 10^{-17} seconds, a mass of 10^{-31} kg, a time of 10^{-24} seconds, a distance of 10^{-15} metres, a distance of 10^{10} light years.

We cannot really comprehend the magnitude of these quantities and yet . . .

10^{55} kg is the estimated mass of the Universe.

9×10^{-31} kg is the mass of an electron at rest.

10^{17} seconds is the estimated lifetime of the Universe.

10^{-15} metres is the diameter of an atom.

10^{-24} seconds is the shortest time interval studied, although direct methods of doing this are unknown.

10^{10} light years is the extent of the known part of the Universe.

A further comparison of large and small follows in the table.

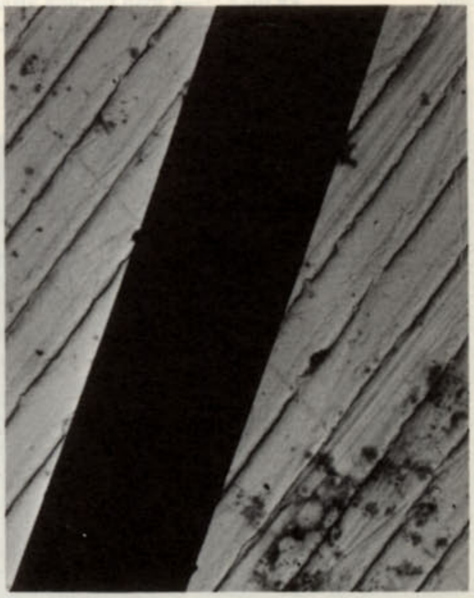


Fig 2.5 This microphotograph of a tungsten wire of nominal diameter 5μm was taken against a grid with 2600 lines/centimetre. What is the degree of magnification? What was the actual diameter of the wire?

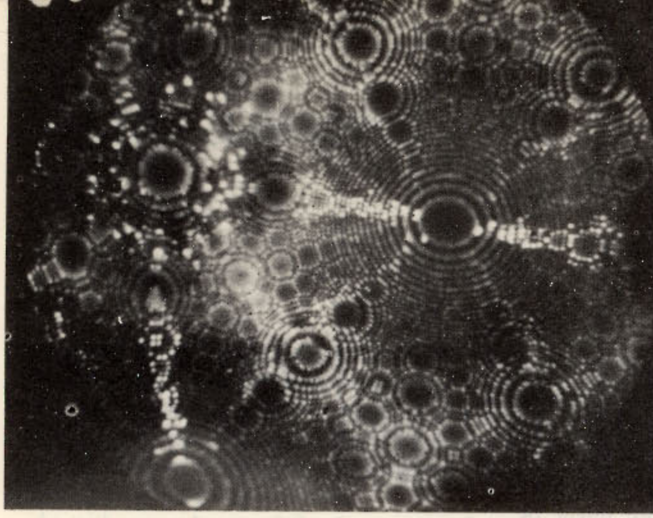
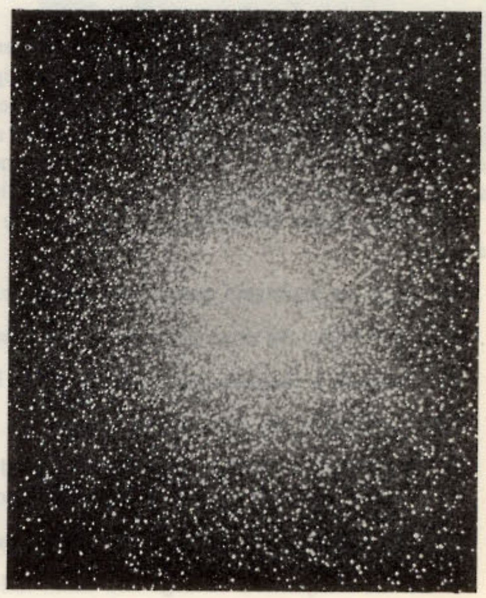


Fig 2.6 A Field Ion Emission Microscope picture of the point of a tungsten needle. Magnification is about 2 million times. Each spot of light represents an atom. What does this tell us about the structure of the material of which the needle is composed?

PHYSICAL QUANTITY	COMMON UNIT IN THE LARGE-SCALE WORLD	SCALE IN THE SUBMICROSCOPIC WORLD
Length	Metre—length of a normal adult pace; average adult height is about 2 m	1 Angstrom = 10^{-10}m— size of an atom
Time	Second—the swing of a pendulum; time for a pulse beat	Most particles 'live' for about 10^{-23} s
Mass	Kilogram—about two pounds. Your head has a mass of about 5 kg	Mass of an electron = 9×10^{-31} kg
Energy	Joule—energy required to pick up a pen from the floor	1 eV (electron volt)—now superseded; 1.6×10^{-19} J. Air molecule has 0.04 eV accelerated proton up to 3×10^{10} eV $= 4.8\times10^{-9}$ J
Speed	Metre per second; normal walking pace	Speed of light = 3×10^{8} m s^{-1}
Charge	Coulomb (now more correctly ampere second) the total charge of the particles in a full stop	Charge on the electron $= 1.6\times10^{-19}$ C $= 1.6\times10^{-19}$ A s
Spin	kg m^{2} s^{-1}—a man turning around slowly	Spin of photon = 10^{-34} kg m^{2} s^{-1}

Fig 2.7 The spiral nebula (left) is ten thousand times further away than the globular cluster in our own galaxy (right).



3: UNCERTAINTY IN MEASUREMENT

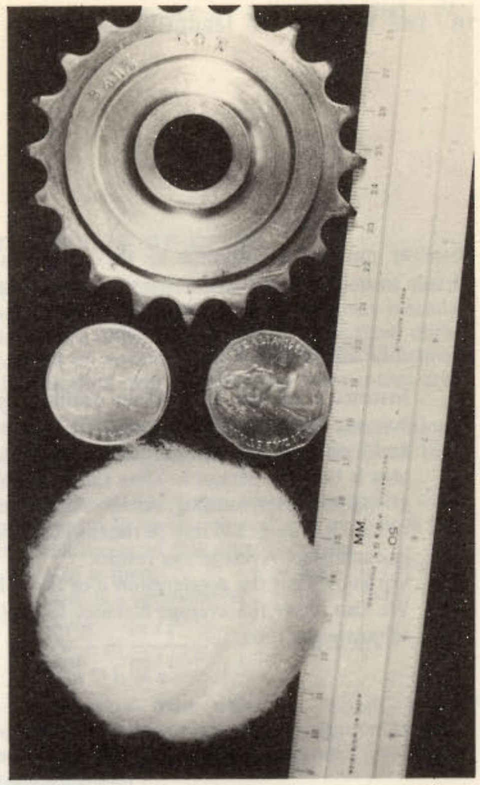


Fig 3.1 *What are the diameters?*

In making any measurement we must accept the fact that we must allow for some variation in the measured quantity. This variation may result from the person making the measurement, from the limitations of the measuring instrument, or from the limitations of the quantity being measured. If we consider this last source of variation, perhaps this can be made clearer in the following manner. Consider the page you are reading at the moment—how long is it?

You might try to measure the length with a ruler and if you did then you could give an answer to the nearest millimetre. If you used a travelling microscope to sight the edges of the paper, then your measuring instrument would be more precise, but another difficulty would arise—where exactly is the edge of the paper? The edge that looks so definite when viewed with the naked eye appears quite fuzzy and indistinct under the microscope. To take a more obvious example, what is the diameter of a 50c coin, or one with a milled edge, for that matter? A tennis ball? Try to make some measurements from fig 3.1. The scale is given. Discuss your difficulties.

To measure the thickness of a sheet of paper you might use a micrometer and a single sheet and take several measurements to obtain an average.

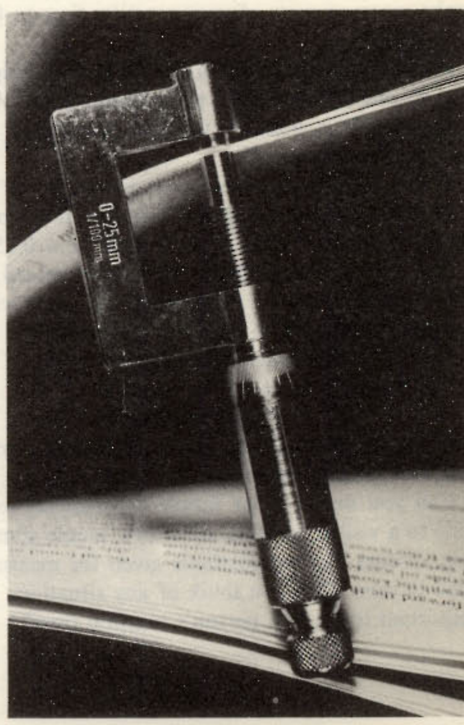
EXPERIMENT

Take a single page of this booklet. Use a micrometer to measure its thickness. How reliable is this measurement? What factors contribute to its unreliability?

Now take 20 pages of the same booklet and measure the thickness of the 20 pages. Divide by 20 to get the thickness of one page. Compare this with your first value. Which is more reliable? Why?

Can you suggest how to measure the thickness of a sheet of paper given a rule with only two marks on it, one metre apart (and enough paper). How do you think this would compare with your first method? The thickness may also be found by using an indirect optical method, and this will be discussed later.

Fig 3.2 *A micrometer.*



The measuring instrument itself limits the degree of accuracy of any measurement. If the instrument is faulty in any way then the measurements may be unreliable, i.e. repeated estimates of the same thing may vary. Alternatively, if the instrument is not faulty, but simply out of adjustment, there may be a consistent, predictable error. If you are aware of it you can make allowance for it.

The person taking the measurements is a little less predictable. Statistically we can allow for the variation in particular measurements made by an observer if we have previously examined the performances of a large random sample of similar observers under similar conditions.

To illustrate this, consider the term, *reaction time.* This is the time interval between the occurrence of a given stimulus and the observer making a response to this stimulus. Use one of your friends to help you test your reaction time to a visible stimulus.

EXPERIMENT

Instruct a partner to hold an ordinary school ruler vertically between thumb and forefinger at the 12″ mark. Hold your thumb and forefinger adjacent to, but not touching the zero mark. The idea is that the person holding the ruler should drop it without giving you prior warning. As soon as it is dropped you must try to stop it falling. Do this 50 times and record the distance fallen in each case. Average the results.
We know that the acceleration a of the rule is $g \simeq 10\ \mathrm{m\,s^{-1}}$. We also know the average distance fallen (s). We can therefore calculate the time:

$$s = 1/2\,at^2$$
$$t = \sqrt{2s/a}$$
$$\text{If } s = 20\text{ cm, } t = \sqrt{\frac{2\times 20}{10\times 100}} = \sqrt{\frac{4}{100}} = \frac{2}{10}$$

i.e. your reaction time is one fifth of a second.

Using a slightly more sophisticated measuring device, you can make measurements of your reaction time to 8 or even 16 significant figures.

A computer can perform additions or multiplications at a fixed rate. If one writes a short programme (a set of instructions to the computer) it is possible to start "counting" from zero and to continue until stopped.

If the interval between each count is 10^{-4} seconds and the computer has counted up to 2568 when it is stopped, then the time interval would be 2568×10^{-4} seconds = 0.2568 seconds.

The "start" button lights up when pressed. Hence the person whose reaction time is being measured waits with his finger on the 'Instant Stop' button. As soon as he sees the light he pushes the button. The time taken is then printed out.

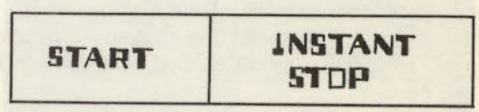


Fig 3.3

Which would be (a) more accurate?
(b) more reliable?

One reading from the computer of 0.2568 seconds.
The average from the ruler experiment of, say, 0.25 seconds.

Can you devise other simple experiments to test your reaction time to a visible, audible, olfactory, tactile stimulus? Before doing this, discuss the two methods given for measurement of reaction time in class. Can you think of any situations in which it may be important to know a person's reaction time?

Translated units

Note that in the experiment with the ruler we actually measured *time* as a length, i.e. the ruler was a transducer. We can also work the other way: 'the house is five minutes from the shops', 'London is 24 hours from Sydney'. (And what is a *parsec*, for that matter.)

So for a thought exercise: If you know all the standard physical data (the speed of light and sound, the density of water, etc.) how would you describe:

0.001 m, 1.0 m, 1000 m without mentioning a unit of *length*,

0.001 kg, 1.0 kg, 1000 kg without mentioning a unit of *mass*,

0.1 sec, 1.0 sec, 60 sec without mentioning a unit of *time*?

There is another thought exercise (needing pencil and paper as well) in the side column.

ERRORS

In any real scientific experiment we usually start with some knowledge of the final result. The measurements taken simply decrease the uncertainty in the answer, although some degree of uncertainty must always remain regardless of the ability of the observer or the sophistication of the measuring device. Any measuring instrument disturbs that which it is intended to measure. Nothing can be measured in its undisturbed state. In order that we see an object it must reflect or emit light. Is this reflection just like tennis balls bouncing off a wall or does the light interact with the reflecting surface? (If the latter is true can we say that the object reflecting the light is undisturbed ? This is further discussed in a later section.)

Try to measure the temperature of a thimble full of hot water with an ordinary laboratory thermometer. Do you think your result is valid? Accurate? Reliable? Think of the meters used to make measurements in electrical work. Do they upset the system being measured? Can we allow for this?

Having made our measurements, we must indicate the estimate made, and the degree of uncertainty, e.g. for a length measured with a metre ruler, with the smallest divisions being millimetres, we might estimate a length to be 2.55 cm ± 0.05 cm or (2.55 ± 0.05) cm. This simply indicates that the measured length may be as small as 2.50 cm or as large as 2.60 cm.

It is impossible then, to 'measure' anything directly. As we have said before, measurement involves counting and comparison. We simply compare the quantity under observation with some convenient, established standard quantity of the same nature.

If we examine the statement "the average length of ten fingers is ten centimetres", we see that the word "ten" has been used in two quite distinct ways. In the phrase "ten fingers", it represents the natural or counting number. Provided that a person has ten normal fingers and has not suffered an amputation of any of them, then the

THE UNITS OF CZINK

The ancient civilization of Czink was very advanced scientifically, but for various reasons adopted a different set of basic units from ours. Instead of working from length, mass and time, they worked from acceleration, momentum and force.

Their units were the *down* (for acceleration) the *move* (for momentum) and the *push* (for force).

Instead of measuring time in seconds, they measured it in *moves per push*. We can check the reasonableness of this dimensionally:

***push* is force** $F = ma = m\,l\,t^{-2}$

***move* is a momentum** $= mv = m\,l\,t^{-1}$

$$\text{so } \frac{\text{move}}{\text{push}} = \frac{m\,l\,t^{-1}}{m\,l\,t^{-2}} = \frac{1}{t^{-1}} = t(\text{time}).$$

What were their units for length, mass, velocity, kinetic energy, density, (go on as long as you like—you will find some intriguing relationships).

use of "ten" in this instance implies an exact count. There are exactly ten fingers, not 10.001 or 9.999 regardless of the measuring technique used to arrive at this number. The second use of the word "ten" in the phrase "ten centimeters" is always an approximation. Here we are dealing with a defined standard unit, and to measure the length of a finger involves comparison with that standard.

Since it is theoretically impossible to express any length to infinite accuracy (except the one which provided the original standard), then the word "ten" used in the phrase "ten centimeters" must be an approximation.

When we know that the answer to a problem must be a whole number, we can sometimes eliminate small errors in measurement due to actual technique or slight variation in the objects being measured. As an example, banks count coins by first separating them in a sorting machine and then weighing them. To find the total number of a particular coin, the mass of the unknown number of coins is divided by the mass of one coin (how do you think the latter would be obtained—simply by selecting a coin at random and weighing it?). The bank knows that the answer must be a whole number and they therefore approximate the answer to their calculation *down* to the nearest whole number. (Why *down?*) This is one place where the "approximation" is more accurate than the calculated answer—at least for the bank's purposes (a purist may argue that a worn coin is not a whole coin, but as far as legal tender is concerned they are worth the same.)

More often, however it works the other way. We make measurements of quantities and manipulate these values by adding, subtracting, multiplying and dividing them. The result cannot be more accurate than the *least accurate* of the original data. The accumulated error can be calculated and the following shows you how this can be done:

With a metre rule, we can measure or estimate to one half of a millimetre at best, hence a measurement like (52.35 ± 0.05) cm would contain four significant figures.

An error that is actually involved in the measuring process is an *absolute error*. In this case the absolute error is 0.05 cm. How would you state (2.55 ± 0.05) cm in the M.K.S.A. system of units? Would this change the accuracy of the result?

Absolute errors do not always prove to be very useful in calculations involving measured quantities, e.g. if we measure the length and breadth of an object and we wish to know the area and the degree of uncertainty in the area, how can we do this?

Moreover, an error of 0.05 cm in 100 cm might not be considered large, but what about an error of 0.05 cm in 0.1 cm? Obviously it is the ratio of the absolute error to the measured quantity that is important in assessing the degree of uncertainty. This ratio is known as the *relative error*, or when expressed as a percentage, as the *percentage error*. The example may clarify how this may be used to calculate errors when measured quantities are manipulated to give a derived quantity, e.g. two lengths giving an area.

1. Subtraction and Addition

For addition and subtraction the final absolute error is obtained by *adding* the *absolute* errors.

If $A = 2.00 \pm 0.01$ metre
and $B = 1.00 \pm 0.01$ metre
then $A - B = (1.00 \pm 0.02)$ metre
$A + B = (3.00 \pm 0.02)$ metre

(If subtraction is involved note that the relative error increases. This has important implications in computer programming—see Appendix V.)

2. Multiplication and Division

For multiplication and division (this includes raising to a power) the *relative* errors must be added.

e.g. taking A and B as above

$$\text{relative error in } A = \frac{0.01}{2.00} = 1/200$$

$$\text{relative error in } B = \frac{0.01}{1.00} = 1/100$$

$$\text{product of } A \text{ and } B = 2.00 \times 1.00 = 2.00 \text{ metre}^2$$

$$\text{relative error in this product} = \frac{1}{200} + \frac{1}{100} = \frac{3}{200}$$

$$\text{absolute error} = \text{relative error} \times \text{product} = \frac{3}{200} \times 2.00$$

$$AB = (3.00 \pm 0.03) \text{ metre}^2$$

For division, the relative errors are again added. Calculations cannot decrease the magnitude of the errors made at the time of measurement. The uncertainty always increases as a result of mathematical manipulation of measured quantities.

Significant figures

When we state the result of a measurement we write down a number of figures, e.g. we might measure a length to be 5.25 cm. This implies that we have a measuring instrument that allows us to estimate to the nearest 10^{-2} cm—we actually measured each of the figures stated. In this case the measurement has been made to three significant figures, the number of significant figures being taken as an indication of the measurements actually made. If the least accurate of the data supplied for a calculation had say, 2 significant figures then the calculated value should not be stated to any more than two significant figures. If data were supplied in the form 5.6 m, 8.9 m, 4.3255 m, this would indicate that needless precision had been involved in the third value. To test this assume that the values were the sides of a rectangular box. Calculate the volume to 2 significant figures using the data

1) 5.6 m, 8.9 m, 4.3255 m
2) 5.6 m, 8.9 m, 4.3 m

Approximations

Before commencing an involved calculation, it is customary and sensible to make an *estimation* of the result. If you intend to use a slide rule then it is essential that you are aware of the order of magnitude of the result. (Very often this is all that is required in many calculations that are done to set up an experiment.)

e.g. if you had the problem $\dfrac{59.52 \times 32.05}{123.2}$

you could say that this was approximately $\dfrac{60 \times 30}{120}$ or 15

Now use logarithms and carry out the calculation. What is the percentage difference between your calculation and the approximate answer of 15?

Computer error programme

Once you understand how errors in measured quantities can lead to greater errors in derived quantities, and you are aware of how the uncertainty in the derived quantity may be calculated knowing the uncertainty in the measured quantities, there is little point in performing such calculations endlessly. Suppose you had to calculate the error in the surface area and volume of a number of rectangular blocks after you had measured their dimensions.

It is a relatively simple matter to write a computer programme to do this. Computers and measurement calculations are firmly wedded in modern day physics. It is difficult to say how physics

EXAMPLES

Quantity $A = (10.0 \pm 0.1)$ metre and quantity $B = (25.0 \pm 0.2)$ metre. Find the absolute, relative and percentage errors in:—

(a) $A+B$
(b) $B—A$
(c) AB
(d) A/B
(e) A^4

(a) Quantity $(A+B) = 10.0+25.0 = 35.0$ metre

absolute error = sum of absolute errors in A and B $= 0.1+0.2$ $= 0.3$ metre

$\therefore A+B = (35.0+0.3)$ metre

relative error in $(A+B)$

$$= \frac{\text{Absolute error in } (A+B)}{(A+B)}$$

$$= \frac{0.3}{35}$$

$$= \frac{3}{350} \approx \frac{1}{120} \text{ (approx.)}$$

percentage error = relative error $\times$ 100

$$= \frac{100}{120} = 0.8\%$$

(b) Quantity $(B—A) = 25.0—10.0 = 15.0$ metre

absolute error = sum of absolute errors in A and B $= 0.1+0.2$ $= 0.3$ metre

$\therefore B—A = (15.0+0.3)$ metre

relative error in $(B—A)$

$$= \frac{\text{Absolute error in } (B-A)}{(B-A)}$$

$$= \frac{0.3}{15}$$

$$= \frac{3}{150} \approx \frac{1}{50}$$

percentage error $= R.E. \times 100$

$$= \frac{100}{50} = 2\%$$

(*continued on page 20*)

could progress if computers were not available to do the tedious calculations often required. The programme given below is in the FORTRAN language. You could write a programme to calculate surface area and volume uncertainties for any shaped body, but we will illustrate the method with a simple programme used for rectangular bodies. (Format statements should continue on the same line.)

```
          PROGRAM UNCERT (INPUT, OUTPUT)
C         TO READ IN LENGTH (ALONG), WIDTH (WIDE),
C         THICKNESS (THICK), AND MASS (BMASS), FOR A
          RECTANGULAR
C         BAR—WITH THE ABSOLUTE ERRORS IN EACH (A, W, T
C         AND B RESPECTIVELY)
C N       IS THE NUMBER OF SETS OF VALUES READ IN
          READ 200, N
  200     FORMAT (I4)
          DO 2 I = 1, N
          READ 201, ALONG, WIDE, THICK, BMASS, A, W, T, B
  201     FORMAT (8F10.4)
C         THE RELATIVE ERRORS ARE NOW CALCULATED
          RA = A/ALONG
          RW = W/WIDE
          RT = T/THICK
          RB = B/BMASS
C         THE VALUES OF SURFACE AREA, VOLUME, AND
          DENSITY   ARE
          SURFA  =  2.0*WIDE*ALONG+2.0*
        1 THICK*WIDE+2.0*ALONG*THICK
          VOL = THICK*WIDE*ALONG
          DENS = BMASS/VOL
C         THE ABSOLUTE ERRORS IN EACH DERIVED
C         QUANTITY ARE NOW CALCULATED
          E1 = (RW+RA)*WIDE*ALONG
          E2 = (RW*RT)*WIDE*THICK
          E3 = (RA*RT)*ALONG*THICK
          E4 = E1+E2+E3
C E4      IS THE ABSOLUTE ERROR IN SURFACE AREA
          E5 = (RA+RW+RT)*VOL
C E5      IS THE ABSOLUTE ERROR IN VOLUME
          E6 = (RB+E4)*DENS
C E6      IS THE ABSOLUTE ERROR IN DENSITY
          PRINT 202, ALONG, WIDE, THICK, BMASS
  202     FORMAT(8H1   LENGTH,F12.4,5HWIDTH,F12.4,9H
          THICKNESS,F12.4,
        1 4HMASS, F12.4,)
          PRINT 203, SURFA, E4
          PRINT 204, VOL, E5
          PRINT 205, DENS, E6
  203     FORMAT(16H SURFACE AREA = , F15.4, 8HERROR = ,
          F10.4)
  204     FORMAT(10H VOLUME = , F15.4, 8HERROR = , F10.4)
  205     FORMAT(10H DENSITY = , F15.4, 8HERROR = , F10.4)
    2     CONTINUE
          END
```

(*continued from page 19*)

(c) Quantity AB $= 10.0 \times 25.0$

$= 250$ metre^2

relative error $=$ sum of *R.E.*'s in A and B

$$= \frac{1}{100} + \frac{2}{250}$$

$$= \frac{5+4}{500} = \frac{9}{500}$$

$$= \frac{1}{50} \text{ (approx.)}$$

absolute error $=$ quantity $\times$ *R.E.*

$$= 250 \times \frac{1}{50} = 5 \text{ metre}^2$$

percentage error $=$ *R.E.* $\times$ 100

$= 2\%$

AB should then be stated as (250 ± 5) metre^2

(d) Quantity A/B $= \dfrac{10.0}{25.0} = 0.4$ (note—no units)

relative error $=$ sum of *R.E.*'s in A and B

$$= \frac{1}{50}$$

percentage error $=$ *R.E.* $\times$ 100

$= 2\%$

absolute error $= 0.4 \times \dfrac{1}{50}$

$= 0.008$

A/B should then be stated as (0.4 ± 0.008)

(e) Quantity $A^4 = 10.0^4 = 10\,000$ metre^4

(or $A^4 = 10.0 \times 10.0 \times 10.0 \times 10.0$)

relative error in $A^4 =$ *R.E.* in A + *R.E.* in A + *R.E.* in A + *R.E.* in A

$= 4$ (*R.E.* in A)

$$= 4 \times \frac{1}{100} = \frac{1}{25}$$

percentage error $= \dfrac{1}{25} \times 100$

$= 4\%$

absolute error $= \dfrac{1}{25} \times 10\,000$

$= 400$ metre^4

A^4 should then be stated as $(10\,000 \pm 400)$ metre^4

You may like to vary the programme, refine it, make it more efficient or more elaborate. If you had, say, 1000 of these calculations to do, it would take you many hours of tedious (and probably only partially correct) calculation. An hour or so writing a programme is the other alternative (providing your programme "runs", i.e. is accepted by the computer and yields sensible results)—the calculations are then done in a matter of seconds. We leave it to you to decide which is the better method.

Consider the following problem:—

Imagine you were given a piece of material of the shape illustrated, such that l is approximately 5 cm, $R \approx 1$ cm, $r \approx 0.8$ cm, $m \approx 50$ g. (It would be better to use M.K.S.A. Units, but we will work this through in C.G.S. Units.) How would *you* proceed to determine the density of the substance?

Suppose we used a vernier to measure l, R and r such that

$$l = (5.00 \pm 0.01)\text{ cm} \qquad \text{relative error} = \frac{1}{500}$$

$$R = (1.00 \pm 0.01)\text{ cm} \qquad \text{relative error} = \frac{1}{100}$$

$$r = (0.80 \pm 0.01)\text{ cm} \qquad \text{relative error} = \frac{1}{80}$$

$$\begin{aligned}\text{Volume} &= \pi l(R+r)(R-r)\\ &= \pi(5.00)(1.00+0.80)(1.00-0.80)\\ &= \pi \times 5.00 \times 1.80 \times 0.20\\ &= 1.80\pi\\ &= 5.65\text{ cm}^3\end{aligned}$$

Absolute error in

$(R+r) = (0.01+0.01)$ cm $= 0.02$ cm

Absolute Error in $(R-r)$ also $= 0.02$ cm

$$\text{Relative error in } (R+r) = \frac{0.02}{1.80} = \frac{1}{90}$$

$$\text{Relative error in } (R-r) = \frac{0.02}{0.20} = \frac{1}{10}$$

$$\text{Relative error in } (R+r)(R-r) = \frac{1}{90} + \frac{1}{10} = \frac{1+9}{90} = \frac{1}{9}$$

$$\text{Relative error in } l(R+r)(R-r) = \frac{1}{9} + \frac{1}{500} \approx \frac{1}{9}$$

$$\therefore \text{ Relative error in volume} = \frac{1}{9}$$

$$\%\text{ error in volume} = \frac{100}{9}\% \approx 11\%$$

Not a particularly accurate result!

It would almost certainly be better to use displacement of water in a measuring cylinder to find the volume—it would be quicker and no less accurate.

A triple arm balance could be used to find the mass and then density could be calculated from mass/volume.

EXPERIMENT

Take at least ten sheets of quarto (or foolscap) paper. Estimate a length of 0.1 metre (10 cm) and place two pencil dots on each sheet a distance apart representing your estimate. Orient the paper differently in each case and do not use a ruler for reference. (Why should you change the position of the dots on the paper, i.e. turn the sheet around?)

Now check your estimates with a ruler (to the nearest millimetre). You may obtain results like $(0.950 \pm .005)$ metre, etc. Average your results using the rules you have been given. How good an estimate did you make? Try asking your parents to perform the experiment (without first allowing them to see a ruler with the required length marked on it). Are their final estimates as good as yours? Suggest a reason. What in fact, does this experiment show?

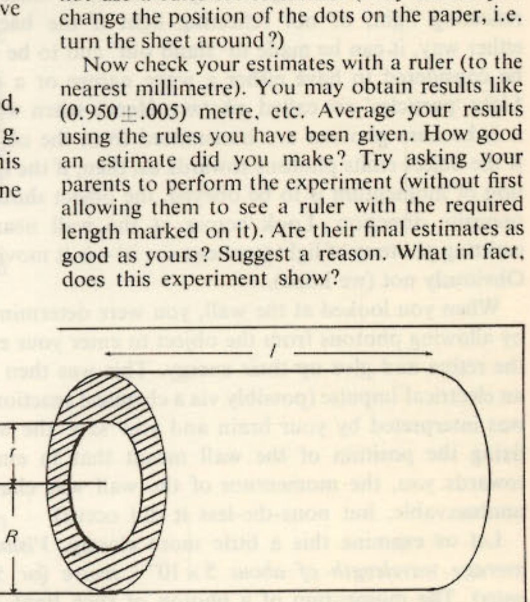


Fig 3.4

EXPERIMENT

If you were given an object, rectangular in shape, weighing approximately 1 kg and measuring approximately 0.2 m × 0.1 m × 0.05 m, what instruments (from the following list) would you use to calculate the density of the material in the object? Explain your choice. Include any other methods if none of those below is suitable.

Possible Error	*Possible Error*
Metre rule ($\pm$0.05 cm)	Beam balance ($\pm$0.005 g)
Micrometer ($\pm$0.001 cm)	Spring balance ($\pm$1.0 g?)
Vernier ($\pm$0.01 cm)	Deca-gram balance ($\pm$0.1 g)
Travelling microscope ($< \pm$0.0001 cm)	Electronic balance ($\pm$0.001 g)

The possible error depends on the scale reading limitation and errors introduced by moving parts, etc.

MEASUREMENT OF THE VERY SMALL—UNCERTAINTY

As physicists continue to probe the nature of matter, they eventually come to the measurement of quantities that cannot be specified with anything like the certainty that we can ascribe to, say, the mass of a football, or the position of an object relative to them.

In order that the object may be seen, it must be capable of reflecting light, or not reflecting light if the background does; either way, it can be made to 'stand out' and to be seen. Light can be considered to have either a wave nature or a particle nature. Light 'particles' are called *photons*. Hence when we see an object, it is because photons are transmitted from the object to our eye. If the object emits photons towards us, then, if the law of conservation of momentum is to be obeyed, the object should recoil in the opposite direction. Look across at the wall nearest you—it is emitting photons of light (you can see it)—is it moving backwards? Obviously not (we hope).

When you looked at the wall, you were determining its position by allowing photons from the object to enter your eye, impinge on the retina and give up their energy. This was then converted into an electrical impulse (possibly via a chemical reaction). The impulse was interpreted by your brain and you 'saw' the wall. The act of fixing the position of the wall meant that in emitting photons towards you, the momentum of the wall was changed (this was unobservable, but none-the-less it did occur).

Let us examine this a little more closely. *Visible light has an average wavelength of about* 5×10^{-7} *metre* (*or* 5000 *Angstrom units*). The momentum of a photon of such light is given by the formula—(units used are M.K.S.A.)

$$\text{(momentum) } p = \frac{h \text{ (Planck's constant)}}{\lambda \text{ (the wavelength)}}$$

h has the value 6.63×10^{-34}Js (joule second).

Hence the momentum of a photon would be—

$$p = \frac{6.63\times10^{-34}}{5\times10^{-7}} \text{ sN (second newton)}$$

$$\approx 1.3\times10^{-27} \text{ sN}$$

Imagine that the wall had a mass of 200 kg (does this seem reasonable?).

The emission of one million, million, million photons (10^{18} photons) would give the wall a backward velocity of—

$$v = \frac{1.3\times10^{-27}\times10^{18}}{200} \text{ ms}^{-1}$$

$$= 0.65\times10^{-11} \text{ ms}^{-1}$$

Small wonder that you are not able to see the wall move.

Fig 3.5 Metallic crystals. See if you can find out how these crystals have been made visible?

Now let us consider that we wished to determine the position of a single electron (mass = 9×10^{-31} kg). To do this we have to "reflect" light (photons) off the electron. The photon we discussed before has a momentum of the order of 10^{-27} sec. Newtons, the electron has a mass of the order of 10^{-30} kg, hence the recoil velocity of the electron could be quite high (estimate what it might be).

So, on the scale of subatomic particles, we set ourselves a problem—to look at a particle and hence determine its position, we have to upset its momentum. To determine the momentum of the particle, we have to upset its position. One measurement can only be made to any accuracy at the expense of the other. This fact was recognised by a man called Werner Heisenberg who stated it in the following way—

$$\Delta x \times \Delta p \geqslant h$$

This simply means that we cannot measure momentum and position at the same instant with unlimited accuracy.

Δx is the uncertainty in the measurement of position

Δp is the uncertainty in the measurement of momentum

h is, again, Planck's constant.

A similar relationship exists between energy and time.

$$\Delta t \times \Delta E \geqslant h$$

As an example, say an electron has a speed of 500 ms^{-1} accurate to 0.01 %. With what accuracy can we specify the position of such an electron?

$$p = mv = 9 \times 10^{-31} \times 5 \times 10^{2}$$
$$= 4.5 \times 10^{-28} \text{ s N}$$

The uncertainty is stated as 0.01 % of this quantity

$$\Delta p = 4.5 \times 10^{-28} \times 10^{-4}$$
$$= 4.5 \times 10^{-32} \text{ s N}$$

The minimum uncertainty in position, calculated using the uncertainty principle is hence—

$$\Delta x = h/\Delta p$$
$$= \frac{6.6 \times 10^{-34} \text{ Js}}{4.5 \times 10^{-32} \text{ sN}}$$
$$\approx 1.5 \times 10^{-2} \text{ metre}$$
$$= 1.5 \text{ cm}$$

It is rather hard to imagine the electron as a tiny dot if you cannot measure its position to any greater accuracy than this. Look back at the table to see the size of an average atomic nucleus—compare this with the uncertainty in position (and hence really "size") of the electron.

The uncertainty principle is really important, then, in the physics of the very small.

For comparison let us take a more familiar object—say a 50 kg boy running a hundred metres in ten seconds (quite a feat, but the boy's difficulty in performing it is more than offset by the ease of calculation these figures afford us). Assume the uncertainty in the measurement of this feat is again 0.01 %.

$$\text{Momentum } (p) = mv$$

$$= 50 \times \frac{100}{10} = 500 \text{ sN}$$

$$\text{Uncertainty } \Delta p = 500 \times 10^{-4}$$
$$= 5 \times 10^{-2} \text{ s N}$$

$$\Delta x = \frac{6.6 \times 10^{-34} \text{ Js}}{5 \times 10^{-2} \text{ sN}}$$

$$= 1.1 \times 10^{-32} \text{ metre}$$

Compare this again with the size of an atomic nucleus (10^{-15} metre).

This uncertainty in position is unmeasurably small, hence it is of no importance in ascertaining the position of the boy.

The uncertainty principle fades with insignificance in the macroscopic (large scale) world, but it is an all important factor limiting the accuracy of measurement in the subatomic world.

G. Gamow in his book *Mr. Tompkins in Wonderland* has speculated what our world would be like if the value of Planck's constant were much larger than it is now.

The uncertainty effects experienced in our dealings with subatomic particles would become important in the macroscopic world. Imagine trying to hit a baseball, drive a car, or do any of those familiar things we take for granted.

4: TRANSDUCERS

A transducer is a device that converts a measurable quantity into something more conveniently measured or used.

As a simple example of a transducer consider the ordinary mercury-in-glass thermometers. Increase in temperature is measured in terms of variations in length of a mercury column. The temperature variation is more conveniently displayed as a length.

Most transducers convert measurable quantities into some sort of electrical signal as this is most readily transmitted, e.g. the sensors on a satellite orbiting the earth may be measuring such things as micrometeorite activity, ultra-violet radiation intensity, temperature, magnetic field strength, etc. All of these variables are transformed into electrical signals by transducers and then transmitted to earth where further transducers may convert them into a form suitable for study.

Think of the simple act of looking at a light globe. An electrical current is transformed into electromagnetic radiation which we "see" as visible light. Our eyes ultimately enable an electric signal to be sent to the brain. This is probably compared in some way with the "memory" of past impulses and interpreted as a "light globe".

Is the speedometer on a car a transducer? What about the petrol gauge?

Transducers are used widely even in the home. You have several of them in your own house. The T.V. set, radio, radiogram, all act as transducers. How many other examples can you think of? Make a list, stating the function of each one.

In the next section we shall discuss some transducers. When you have completed the section examine the instruments discussed to see how many can be related to transducers, i.e. they either employ transducers or are themselves transducers.

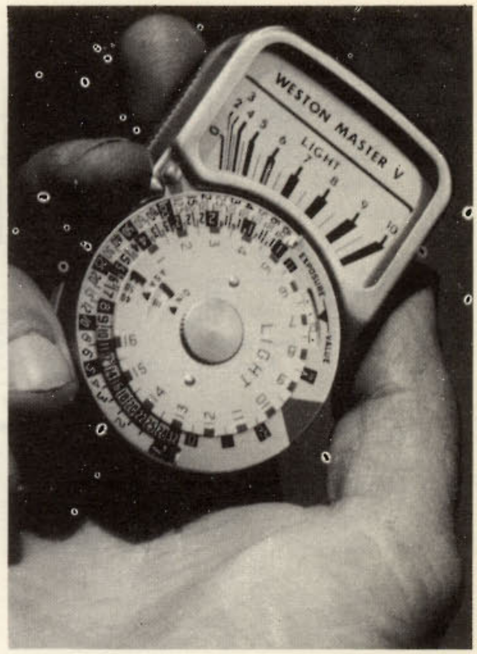


Fig 4.1 A transducer . . . or two?

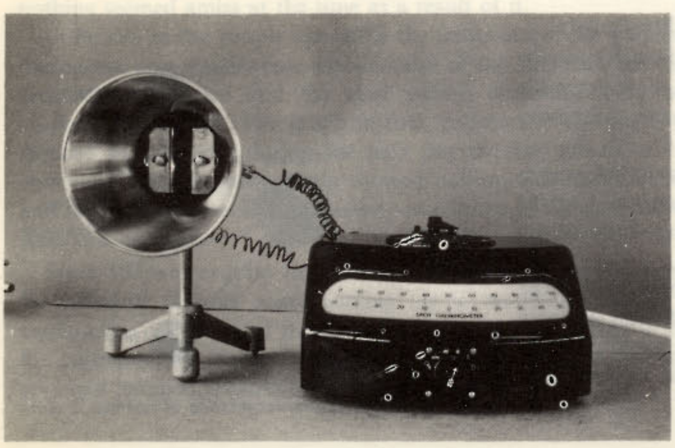


Fig 4.2 See if you can find out what this combination of instruments is used for. How many transducers are there in use here?

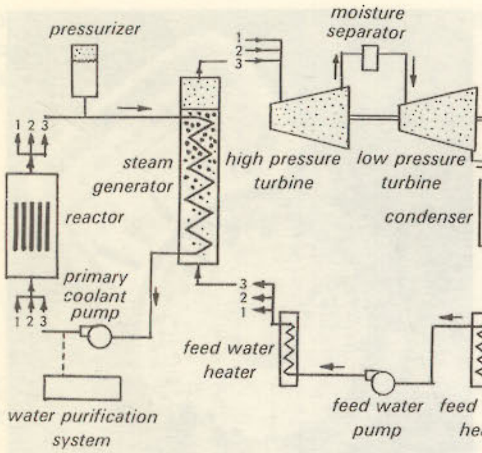
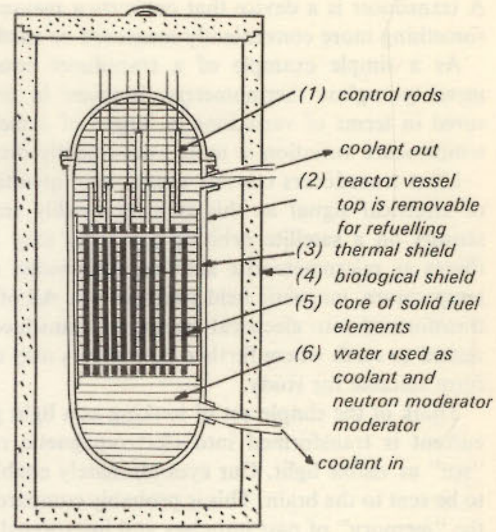


Fig 5.1 Layout of a typical atomic power plant.



A nuclear reactor in an atomic power station may perhaps be considered to be a transducer (is it?). The fission (splitting) of the nuclear fuel into lighter elements causes an enormous release of energy, mainly in the form of the kinetic energy of the fission products. The total mass of the products is less than the mass of the original parent atom. The mass defect or difference in mass has been converted into energy. Einstein's mass-energy equation can be used to calculate the total energy released per fission.

$$E = mc^2$$

The layout of a nuclear power plant could look like fig 5.1.

A block diagram of the reactor (fig 5.1(b)) indicates how the reactor functions. The control rods, when lowered, slow down the reaction, and the water acts both as a coolant (to convey heat away from the reactor) and as a *moderator*.

A moderator is a substance that slows down neutrons released in the fission process, converting their energy into heat, and at the same time increasing the possibility of their being captured by atoms of the fuel and hence causing further fission.

For a reaction to be controllable and self-sustaining, *one* of the product neutrons should go on to cause a further fission. If more than one causes a further fission the chain reaction could result in such a large release in energy in a short time that an explosion could result.

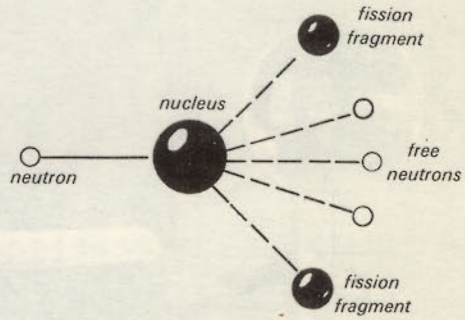


Fig 5.2 Schematic representation of nuclear fission.

It is in the control of the energy release of reactors that measurement is important. Devices that keep a check on the number of neutrons released in the pile continuously monitor the state of the reaction. *Thermo-couples* keep a record of the temperature within the pile. (A thermo-couple is a device that converts heat energy into electrical energy.) *Radiation counters* check the state of the coolant material. If one of the fuel containers should crack and release radioactive material into the coolant, pollution of the plant could result. This should never happen and when you think that all of the safety devices mentioned above are installed in triplicate at least, it might seem that accidents are just not possible,

This unfortunately is not so, and one example may indicate to you just how important it is to know what you are measuring when you work with an instrument that gives a reading on a dial some distance from where the instrument itself is actually sited.

This particular accident occurred at Windscale Pile No. 1, a military reactor in England. This reactor was a little different in design from that shown in fig 5.1. The moderator in this instance was graphite, and the coolant was air. The air was drawn through the pile and, after being checked and filtered, expelled through large smoke-stacks. Hence any release of radioactive substance could possibly result in pollution of the countryside.

During normal running of the reactor, the graphite tended to swell and store some of the heat energy from the pile. This stored energy was released at a later time, the release being effected by shutting down the reactor and slightly heating the graphite. This had the effect of triggering the release of the stored heat.

At the start of one of these routine releases of heat, the monitoring devices were checked to ensure that they were operating properly. Defective thermo-couples were replaced and the procedure commenced. Initially the triggering heat applied did not seem to have the desired effect and so a second heating of the graphite was tried. This was done a little faster than routine called for, but nothing seemed amiss at the time as a result of it.

Unknown to the people operating the treatment of the graphite, during the heat application one or more of the fuel elements had seriously overheated and the steel jackets enclosing them had cracked and melted. The uranium fuel, exposed to the air, commenced to burn and during the next day the fire spread to 150 channels. The pile instruments showed nothing drastically wrong and only a slight rise in radioactivity was noted in the expelled air used as a coolant.

When the temperature rose to a level that caused some concern the operator allowed more cooling air to circulate. Instead of cooling the pile this had the effect of increasing the extent of the fire raging in the reactor core. The instruments used to scan the core had jammed because of the intense heat.

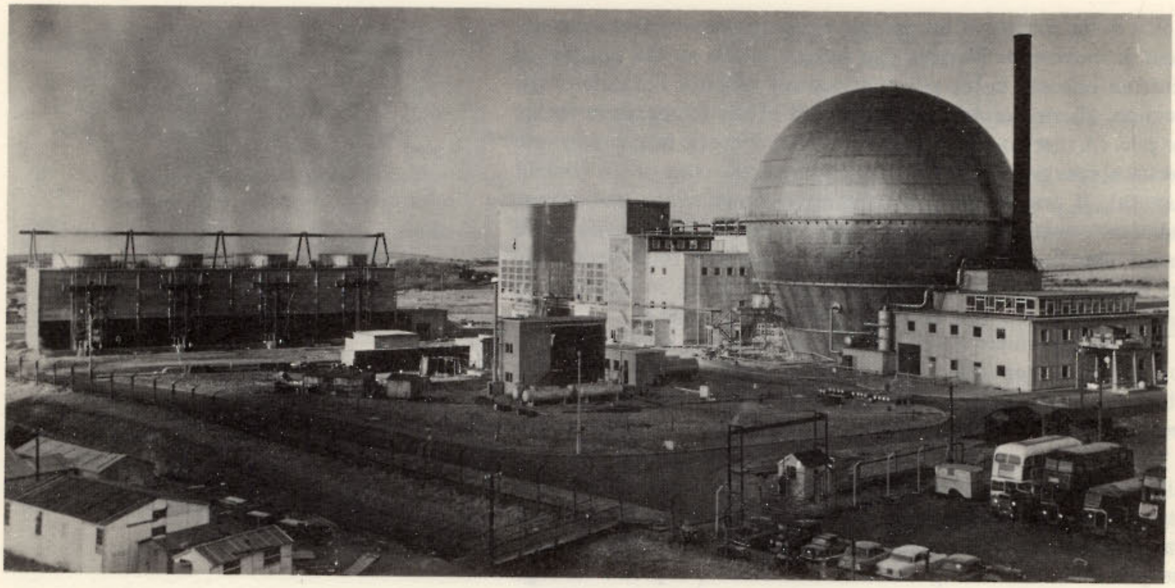


Fig 5.4 The Windscale gas-cooled reactor.

When the alarm was finally raised it was decided to cool the pile using water—a particularly dangerous procedure, but carried out without incident. During the period the pile had been out of control large amounts of radioactivity had been released into the surroundings (for two whole days). The total radioactivity released was estimated to be about one-tenth of that of the atomic bomb dropped on Hiroshima. It had a drastic effect on farming in the area and probably this effect still persists to some degree. This accident occurred in 1957—a totally safe reactor still does not exist.

The point to be learned from this was that although monitoring devices were installed, they were not distributed throughout all sections of the pile. The breakdown occurred in a section of the pile that was ineffectively monitored by measuring devices. The coolant air was inadequately checked for radioactivity. Needless to say, the measurement section of this and other atomic piles is now more adequately policed.

To prevent large-scale damage when dealing with nuclear reactors, efficient, reliable measurements of pile activity are essential, but still accidents occur. Up until 1966 there had been at least ten serious reactivity accidents in non-military establishments alone—some resulting in death of the operators. Four reactors had been put out of action by these accidents and never revived successfully.

An insurance company made an estimate of possible damage resulting from the failure of a medium size reactor located near a large body of water about 30 miles from a major city.

"For the three types of assumed accidents, the theoretical estimates indicated that personal damage might range from a lower limit of none injured or killed, to an upper limit, in the worst case, of 3 400 killed and about 43 000 injured.

Theoretical property damages ranged from a lower limit of about one-half million dollars to an upper limit in the worst case of about seven billion dollars. The latter figure is largely due to assumed contamination of land with fission products.

Under adverse combinations of the conditions considered, it was estimated that people could be killed at distances up to 15 miles and injured at distances of about 45 miles. Land contamination could extend for greater distances." (from *Theoretical Possibilities and Consequences of Major Accidents in Large Nuclear Power Plants,* U.S.A.E.C. document "Wash—740").

Having been rather morbid about the consequences of radioactivity release to our environment, we should consider the units in which this radioactivity is measured.

Fig 5.5 Packing radioactive isotopes for shipment. Stringent precautions are taken against leakage of radioactivity.

UNITS OF RADIATION

Radiation is caused by the disintegration of an unstable nuclide. Particles are emitted and a more stable nuclide is left behind.

The fundamental quantity of the radioactivity, therefore, is the number of disintegrations per second in the radioactive material. The unit is the *curie.*

1 curie (Ci) $= 3.7 \times 10^{10}$ disintegrations per second. This is approximately equal to the rate of disintegration of one gram of radium, which was its original definition. Most laboratory sources are of millicurie or microcurie size.

We are often concerned with the result, rather than the source of radiation. This is called *exposure,* and is defined by the ionising effect of a given amount of X- or γ-radiation on a very small mass of air. The unit is the roentgen.

$$\text{Exposure } X \text{ (in roentgen)} = \frac{\text{the ion charge } \Delta q \text{ (in coulombs) produced in a small mass of air } \Delta m \text{ by the secondary electrons produced by X- or } \gamma\text{- radiation}}{\text{mass of air } \Delta m}$$

$$\text{One roentgen (R)} = 2.58 \times 10^{-4}\ \text{C kg}^{-1}$$

The exposure X at a distance l from a radioactive source can be calculated from:

$$X \text{ (exposure, in R)} = \frac{A \text{ (activity, in Ci)} \times \Delta t}{l^2 \times \Gamma}$$

where Γ is the specific emission constant of the nuclide.

From this the *Radiation Absorbed Dose* (RAD) can be calculated:

$$\text{Absorbed dose } D \text{ (in RAD)} = X \text{ (exposure, in R)} \times \overline{F}$$

where $\overline{F}$ is the factor related to the energy of the radiation and the nature of the material.

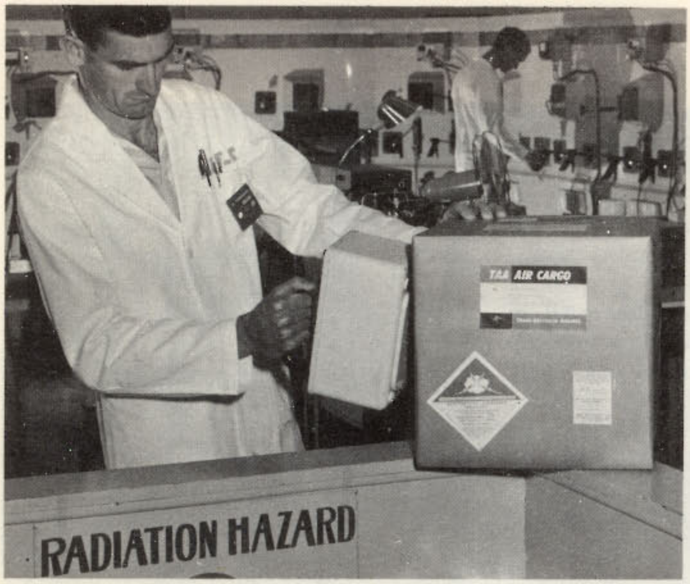


Fig 5.6 This man is checking a package for radioactivity leakage. Note the dosimeter on his breast pocket. It contains a piece of photographic film wrapped in foil. At the end of the day it will be developed, and the extent of the fogging shows whether he has been exposed to abnormal radiation.

The radiation absorbed dose is defined in terms of the radiation energy given to the absorbing material.

$$\text{one RAD} = 0.01\text{J kg}^{-1}$$

The energy of different types of radiation, and therefore the biological effect of them, varies. For example, neutron radiation is ten times as destructive as X- or γ-radiation. Hence a co-efficient, *Relative Biological Effectiveness* (RBE) relates the effectiveness of various types of radiation to that of standard 200 keV X-radiation.

$$\text{RBE} = \frac{\text{Biological effectiveness of a particular radiation}}{\text{Biological effectiveness of 200 keV X-radiation}}$$

Finally, the biological effect is related to the absorbed dose D and the RBE for the radiation in question. It is measured in REM (*Roentgen Equivalent Man*), which is the unit of dose of any ionising radiation which produces the same biological effect as a unit of absorbed dose of standard (200 keV) X-rays.

A dose of 500 REM is considered lethal. See if you can find out more about radiation doses and the safety precautions required in handling radioactive material.

The *Mean Lethal Dose* is that which causes death in 50% of individuals within 20 days if administered to the whole population in 24 hours (about 500-600 REM). This is one measurement that is difficult to obtain accurately due to the lack of willing volunteers to experiment on. We have to rely on information from accidents.

(Have you ever wondered just what a "cutie pie" is—a good looking "bird"? something to eat?—No. It is a common radiation survey meter used to determine exposure levels.)

So if they ever drop the bomb near you and you aren't vaporised by the heat or killed by the blast, at least you will know the units of the radiation that will kill you just as surely, but a little more slowly.

6: TOOLS OF MEASUREMENT

In the direct measurement of length such devices as rules, measuring tapes, vernier callipers, micro-metre screw gauges may be employed depending on the length being measured.

Each instrument is designed for a specific purpose and before using any particular instrument you should always stop to think if it will produce an order of accuracy approximately equal to any others involved in your calculations. There is little point in measuring a length such that you can calculate a volume to an accuracy of 0.01% if, in order to calculate density, mass is measured to 1% accuracy.

With these instruments the length being measured is compared directly with a standard length produced by or incorporated in the instrument. We are not concerned here with how these instruments work, but merely with how, when, and with what validity they may be used.

INDIRECT MEASUREMENT OF LENGTH

If we wish to measure a very great length, e.g. the distance to a star, obviously direct methods fail.

1. Inverse Square Method

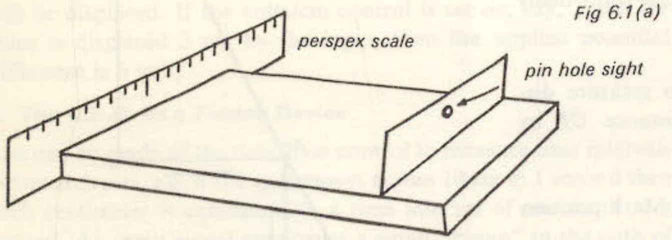
Sometimes, with luminous objects, the intensity of the light received from them may be compared with the intensity from a star a known distance away. Using an *inverse square law* the unknown distance can then be calculated. (Do Experiment S7/3, page 61.)

You may have heard of the word *quasar*. (If not, find out what the term means.) Would the above method be applicable to quasars? Discuss this.

2. Parallax Method

The principle of the parallax methods can be demonstrated in measuring the distance from an observer to a relatively near object. A simple parallax viewer may be used to do this (see *Experiment*).

Fig 6.1(a)



EXPERIMENT

It is essential that a well-defined reference point be used. The reference point should also be many more miles away than the object whose distance from you is to be measured.

Move to a position such that the reference point, object and centre of the parallax scale are all in line.

Mark the position where you stand and move, say, 20 metres (the baseline) at right angles to the line joining reference point, object and centre of the parallax scale.

Stand at the position Q and line up the sight hole, centre of parallax scale and reference point. Without moving the parallax viewer note the scale reading coinciding with the object being measured. Record the apparent displacement of the object from the centre of the scale. Call this x.

The dimensions of the viewer will give you the distance y marked on the diagram.

If the reference point is much more distant than the object then θ is approximately equal to θ_1.

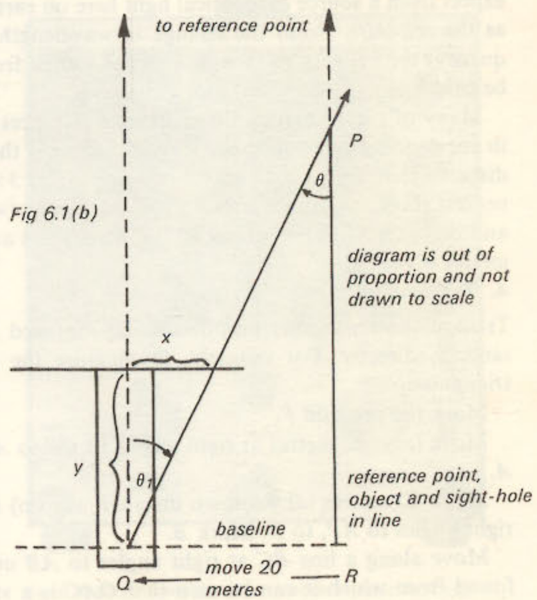
$$\tan \theta_1 = \frac{x}{y}$$

$$\tan \theta = QR/PR$$

$$\text{hence } PR = \frac{y}{x}\, QR$$

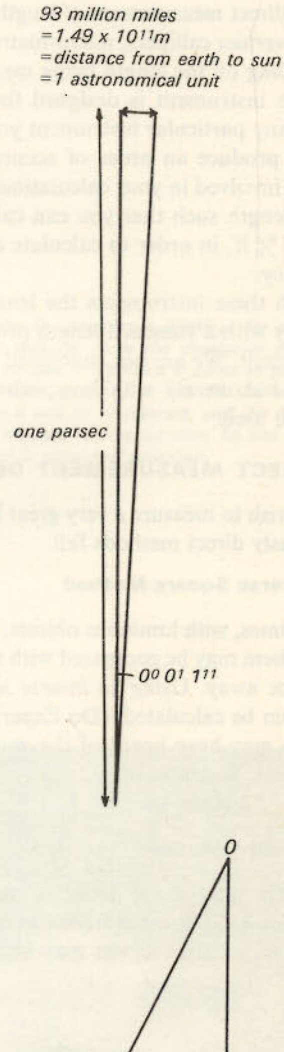
Your Result
$x =$
$y =$
$QR =$
$PR =$

Fig 6.1(b)



The method used here is essentially the same as that used to find the distance of a near star. A distant star is used as the reference point. Once the distance of the near star is established, an extension of the method can be used to find the distance of a far star. The baseline used is the diameter of the earth's orbit. Photographs taken at six-monthly intervals are compared, and the apparent shifts of the distant stars relative to the near stars measured. Although this method works on a base line 186 million miles long, the distances to the stars are so great that the shift can only be measured in *seconds* of arc. A *parsec* (*par*allax-*sec*ond) is the distance at which a body of diameter equal to the radius of the earth's *orbit* would subtend an angle of one second (see fig 6.2). Can you work out a value for the parsec?

Fig 6.2



3. The Doppler Effect

It is generally accepted that nearly all objects in the Universe are receding from each other. It is possible to estimate the speed of recession of luminous objects along a line joining the object to the earth. To do this, use is made of the *Doppler Effect*. You have probably stood by the side of the road as a police car or ambulance sped past with its siren going. You will have noticed that the pitch of the note emitted by the siren changes as the vehicle passes. This change in frequency (and hence pitch) when there is relative motion between source and observer is known as the Doppler Effect.

A similar effect exists for electromagnetic radiation such as visible light. We shall not attempt to go into the theory of this at this stage. Light from a distant star that is receding from us is observed on earth to have a longer wavelength than we would expect from a source of identical light here on earth. This is known as the *red-shift*. From the change in wavelength (and hence frequency) the velocity of recession of the source from the earth can be calculated.

Many of these sources are enormous distances from us. A unit in common usage to measure such distances is the light year—the distance that light would travel in one year at 3×10^8 m s^{-1}. The nearest star to the sun is over 4 light years away (what is it called?) and our galaxy itself is about 60 000 light years across (how many metres is this?).

4. Triangulation

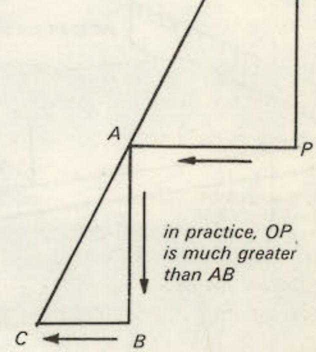
Triangulation methods may frequently be used to measure distances indirectly. For example, to measure the distance *OP* by triangulation—

Mark the position *P*.

Move (say) 20 metres at right angles to *OP* to *A*. Mark position *A*.

Move 40 metres (any known distance will do) in a direction at right angles to *AP*, to *B*. Mark *B*.

Move along a line *BC* at right angles to *AB* until a point *C* is found from which it can be seen that *OAC* is a straight line.



Measure BC.
Using similar triangles,

$$\frac{OP}{AP} = \frac{AB}{BC} \text{ or } OP = \frac{AB \times AP}{BC}$$

All the quantities on the right hand side of this expression have been measured, hence the distance to the object can be calculated.

Can you suggest what are likely to be the main sources of error in this experiment and how they could be reduced? How accurately would you express your answer?

The method of triangulation has wide application. Design an experiment to find the height of an object given a metre rule and a large protractor.

In surveying, the process of triangulation consists of making angular measurements alone. The results are then plotted on to scaled paper using an arbitrary base line that has been drawn to a known scale. The positions of the surveyed points and their relative separation can then be shown accurately on what then becomes a map of the area. The process we have described in detail is used to find one length, given a base line of known length to start with.

Now do Experiment S7/5, page 63

5. THE CATHODE RAY OSCILLOSCOPE

In this unit, we are not concerned with how or why the *Cathode Ray Oscilloscope* (C.R.O.) works, but simply how it may be used for a wide range of measurements. The basic controls are shown in fig 6.4. In our applications of this instrument the *Y*-shift (volts/cm) and the time base are perhaps the most important. The volts/cm control determines the displacement of the beam for a given input signal. The time-base control determines how rapidly the spot will move across the graduated screen.

1. The C.R.O. as a Voltmeter

Adjust the time base until the moving spot appears as a continuous line. If a constant potential difference is applied across the *Y* input (with the trace initially in the undisplaced position) then the trace will be displaced. If the volts/cm control is set on, say, 2 and the trace is displaced 3 cm by the input, then the applied potential difference is 6 volt.

2. The C.R.O. as a Timing Device

Use can be made of the time-base control to measure time intervals between events, e.g. if the spot moves across 10 cm in 1 second then each centimetre is equivalent to a time interval of one tenth of a second. An input signal may cause a small "bump" in the path of the spot. If two input signals occur at points 5 cm apart when the time base control indicates that the beam is traversing 10 cm in 1 second then the time interval between the two input signals must have been 0.5 second.

Fig 6.4 Basic controls of a cathode ray oscilloscope

(a) We can use this to time a falling object.

As the ball bearing passes between the Al-foil strips the circuit is closed and a pulse appears on the C.R.O. tube. Hence the time taken for the ball bearing to travel a known distance can be calculated. If it starts from rest, then the acceleration due to gravity can be calculated.

The same principle can be used for numerous other applications.

Photo cells could replace the Al-foil to give a more accurate reading.

What is likely to influence the uncertainty in any measurements taken in experiments of this nature? (Can errors arise due to the observer, the event being studied or the measuring instruments? Which of these is likely to be most important? How can the degree of uncertainty in each measurement be reduced?)

(b) Using the experimental set up indicated on figure 6.6, the distance x could be calculated if the speed of sound were known.

The initial noise would result in a pulse on the oscilloscope trace from the microphone, the echo would give a second weaker (why?) pulse. If the time base setting is known, then the time between the sound being made and the echo being received can be found—say it is t seconds.

$$\text{hence distance} = \frac{t \times \text{speed of sound}}{2}$$

(why divided by 2?)

(c) Radar works in a somewhat similar way with electro-magnetic waves used instead of sound and with a much faster setting on the time-base control.

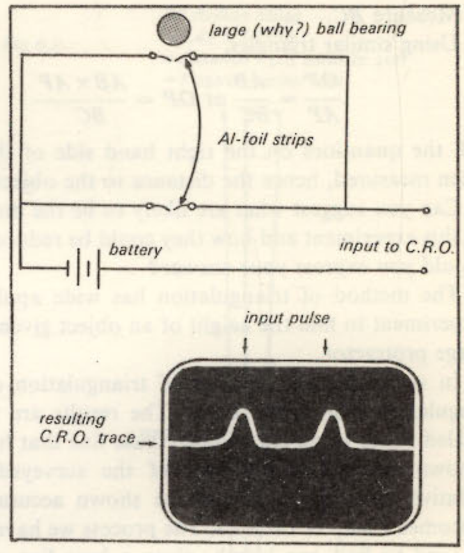


Fig 6.5

3. The C.R.O. Used to Study Waveform

Attach a microphone to the input of the C.R.O. Adjust the time base until the moving spot appears as a continuous line. Talk, whistle, sing into the microphone and observe the trace on the C.R.O. screen. Compare this with the trace from a tuning fork. Play a classical record and observe the trace resulting from this. Now play some of the "mind-destroying clatter" (as it has been called) of some of the poorer forms of popular music—notice any difference?

By correct adjustment of the time base, any wave form can be displayed on the C.R.O.

The effect of mixing two sounds can be seen (the phenomenon of 'beats'—look it up in the library if you are unfamiliar with the term—can be seen as well as heard).

4. The C.R.O. Used to Measure Frequency

(a) This can be done by a direct comparison of a known signal from a signal generator (or 50 Hz A.C.) with an unknown, with the time-base suitably adjusted. (The trace is a signal-time graph.)

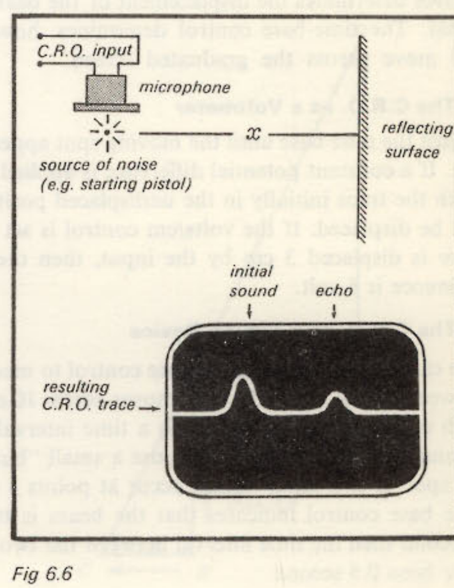


Fig 6.6

First the known signal is used as the input and the wave form noted from the grid on the screen, then the unknown.

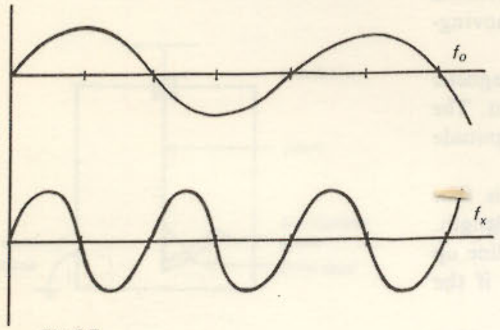


Fig 6.7

From this it can be seen that the unknown signal varies with time twice as rapidly as the known signal.

i.e. $f_x = 2f_o$

(b) A more interesting way of doing this is to put one signal across the *Y* input and the other across the *X* input (normally the time-base operates here).

The traces that result are known as 'Lissajous figures'. If the signals are of the same strength when they are applied across the *X* and *Y* inputs (it may be necessary to amplify the *X* signal first—why?) and the frequencies are almost equal, a 'rocking ellipse' will be seen (fig 6.8(a)). If one frequency is twice the other a rotating 'figure of eight' will be seen (fig 6.8 (b)).

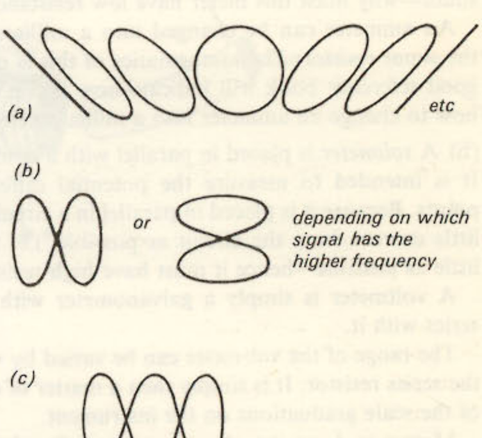
If one frequency is 3 times the other then the result will be as in figure 6.8(c). (Have you seen this anywhere before?)

It is possible to obtain Lissajous figures using two pendula—see if you can find out how.

The C.R.O. can be used to compare the intensity of a signal with that of a known signal. This is virtually the same as using the C.R.O. as a voltmeter.

The uses given here for the Oscilloscope are but a few of many. It can be used to test electrical components for correct operation, to examine phase relationships between signals, analyse signals, etc. You might like to set yourself an assignment to discover just how many different uses the C.R.O. can have.

Fig 6.8



METERS

Look about you and see how many different meters you can see in a day—speedometer, galvanometer (voltmeter, ammeter), chronometer, water meter, etc.

The meters can be roughly divided into two classes, Electrical and Mechanical.

1. Electrical Meters

The most widely used type of electrical meter is the moving-coil galvanometer. Most laboratory voltmeters (to measure potential difference) and ammeters (to measure electric current) are moving-coil meters.

These meters rely on the fact that a coil suspended in a magnetic field will be acted upon by a torque if a current flows in it. The angular displacement of the coil is directly related to the magnitude of the current flowing in it.

The important thing about moving-coil galvanometers is that they have a linear scale. Each scale division is the same length. Hence it is easy to estimate a value if the pointer does not line up exactly with a scale division. This is much more difficult if the meter scale is non-linear.

The meter, like any other instrument, should be designed so that it upsets the quantity that it is designed to measure as little as possible.

(a) An *ammeter* is a galvanometer with a low resistor placed in parallel with it (a shunt resistor). This combination produces an instrument of low resistance that is placed in series in an electrical circuit so that any current flowing in the circuit must also flow through the ammeter, most of it going through the low resistance shunt—why must this meter have low resistance?

An ammeter can be changed into a milliammeter by changing the shunt resistor. The mathematics of this is not difficult and any good reference book will indicate how it can be done. (Find out how to change an ammeter into a milliammeter.)

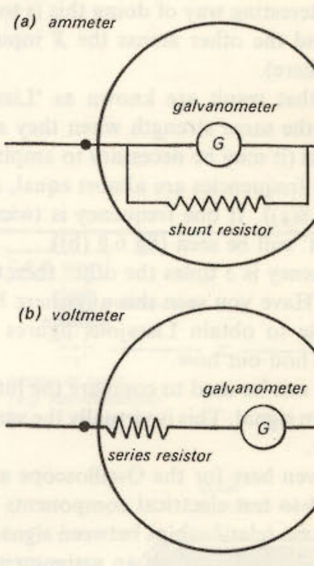
(b) A *voltmeter* is placed in parallel with a component in a circuit. It is intended to measure the potential difference between two points. Because it is placed in parallel in a circuit, it should draw as little current from the circuit as possible, i.e. upset the circuit as little as possible—hence it must have high resistance.

A voltmeter is simply a galvanometer with a large resistor in series with it.

The range of the voltmeter can be varied by varying the value of the series resistor. It is simply then a matter of changing the values of the scale graduations on the instrument.

Meters such as petrol gauges, recording level meters on tape recorders, etc., are simply moving coil galvanometers that have been adapted for a specific use.

Fig 6.9

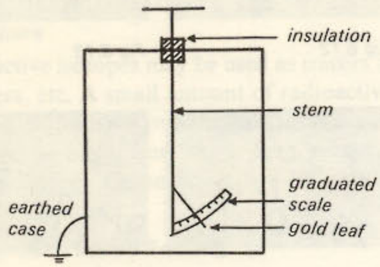


Other Forms of Electrical Meters

There are many other forms of these meters. The diagrams below should be reasonably self-explanatory. Most text books (particularly some of the less recent ones) describe some forms of each of these. Try dismantling an old meter to see how it works (make sure that the meter you dismantle is no longer of any economic value before you try it!).

Fig 6.10 Electrostatic meters.

(a) Gold leaf electroscope. What does it actually measure?



(b) Attracted Disc Electrostatic Voltmeter. When A and C are at different potentials B is attracted to A. The spring suspension causes the pointer to move across a scale. Is this used for high or low P.D. measurement? Is the scale likely to be linear?

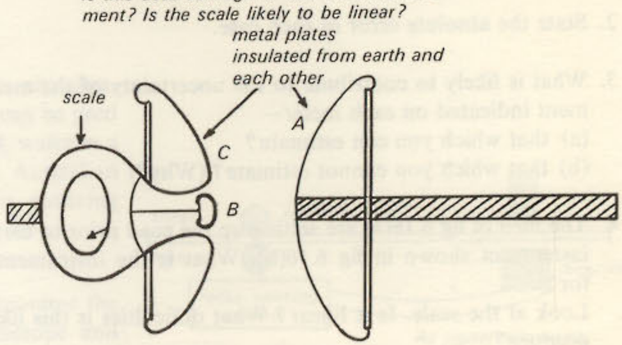
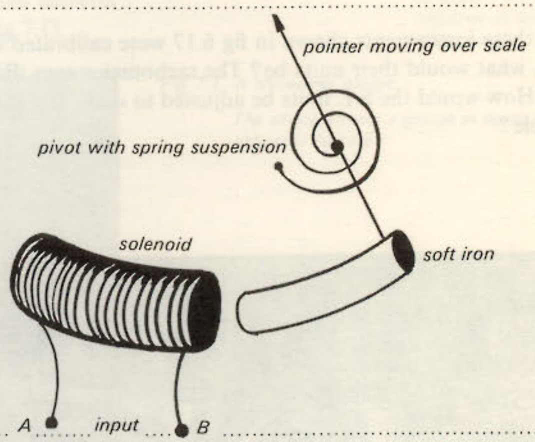


Fig 6.11 Moving Iron Meter.

Soft iron moves into the solenoid against the tension in the spring suspension. Distance moved depends on current in the solenoid.

The diagram shows an electrical current measuring device (A.C. or D.C.?).



2. Mechanical Meter

Perhaps the most common form of such meter would be the water meter. A watch might also be considered as a mechanical meter. This also has a linear scale.

Pressure gauges of various kinds are also mechanical meters in many cases. With all of these meters, what we actually measure is a length the distance a pointer moves across a scale—some of these do not have linear scales.

We assume that the meter has previously been calibrated using an accepted standard and too often we take the reading on the meter at face value without considering:

(a) is the zero correct?

(b) is the calibration correct?

(c) have *parallax* effects, thickness of graduations and pointer ambient conditions been allowed for? (Have you discovered yet how parallax effects cause reading errors?)

QUESTIONS

1. What are the readings indicated by the pointers on the meters in figs 6.12 to 6.15?
2. State the absolute error in each case.
3. What is likely to contribute to the uncertainty of the measurement indicated on each meter—
   (a) that which you can estimate?
   (b) that which you cannot estimate? (Why?)
4. The men in fig 6.16(a) are setting up the road prior to using the instrument shown in fig 6.16(b). What is the instrument used for?
   Look at the scale. Is it linear? What difficulties is this likely to produce?
5. If all these instruments shown in fig 6.17 were calibrated in S.I. units, what would their units be? The tachometer says 'RPM × 100'. How would the S.I. units be adjusted to make the numbers sensible?

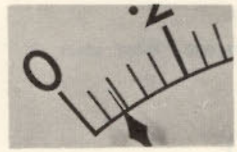


Fig 6.12

Fig 6.13

Fig 6.14

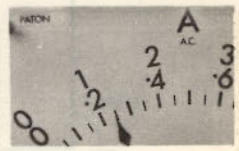


Fig 6.15

Fig 6.16

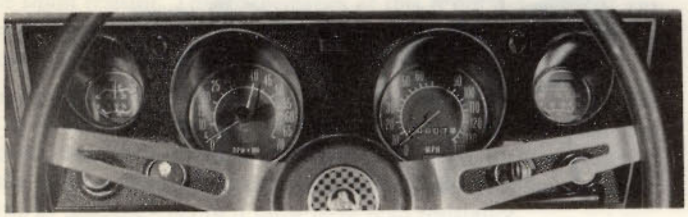


Fig 6.17

BALANCES

The type of balance you use to take a measurement depends entirely on the degree of accuracy required—you may choose to use a spring balance, beam balance, single pan balance. If the relative error in any other measurements you have made was 1/20, say, then it would be pointless to make a measurement of mass any more precise than this.

Our measurement of mass usually depends on a comparison made with a set of standard masses as discussed previously. (How exact are these? How is the comparison made? How accurate is the method of comparison?)

7: SPECIAL TECHNIQUES

Any experiment that you might like to devise is only as good as the measuring techniques that you devise for evaluation of results and initial conditions.

We shall discuss a few special techniques.

1. USE OF RADIOACTIVE ISOTOPES

A. Tracers

Radioactive isotopes may be used as tracers by biologists, chemists, engineers, etc. A small amount of radioactive material can be used as a 'tag' to follow movement of chemicals in the blood, water in a plant, oil in a pipeline, sand from a beach, etc. The Australian Atomic Energy Commission has an excellent booklet outlining many of these uses.

B. Non-destructive Testing

1. Radioactive isotopes may be used to continuously monitor the thickness of sheet material in a rolling mill. The isotope and detector are placed on opposite sides of the sheet and the detector signal is used to determine the spacing of the rollers (fig 7.1).

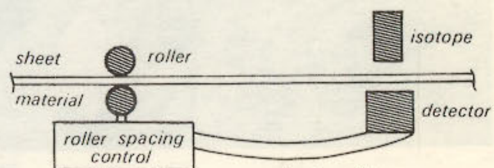
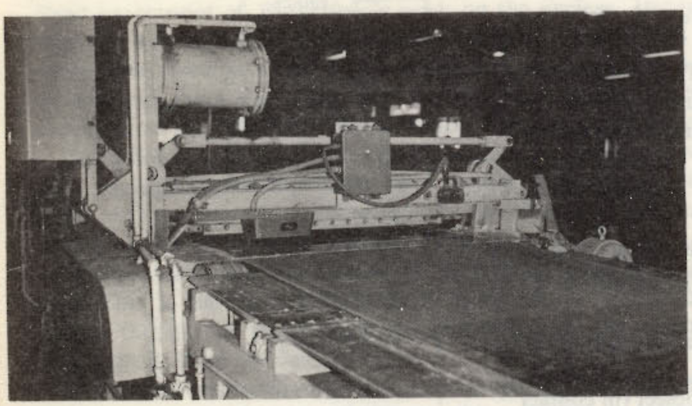


Fig 7.1 A thickness gauge.



The photo shows a gauge in operation in an asbestos factory

Any variation in thickness changes the absorption of the radiation, and this leads to a different signal from the detector and the rollers are adjusted to eliminate this variation.

2. The height of a liquid (say a liquid gas) in a steel cylinder can be measured by the procedure indicated in fig 7.2. When the count rate drops drastically this indicates the level of the liquid.

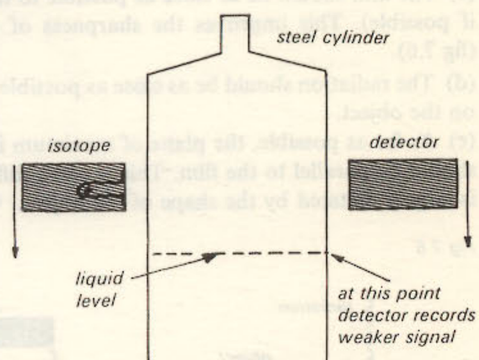


Fig 7.2

3. The thickness of paint on a motor vehicle may be measured by using a back scattering method.

The amount of radiation reaching the detector depends on the nature of the scattering material and consequent absorption in the coating. Hence the thickness can be measured directly (once the detector has been suitably calibrated using a known series of thicknesses of the material to be tested).

Here again the monitoring can be continuous as the objects to be tested move down an assembly line.

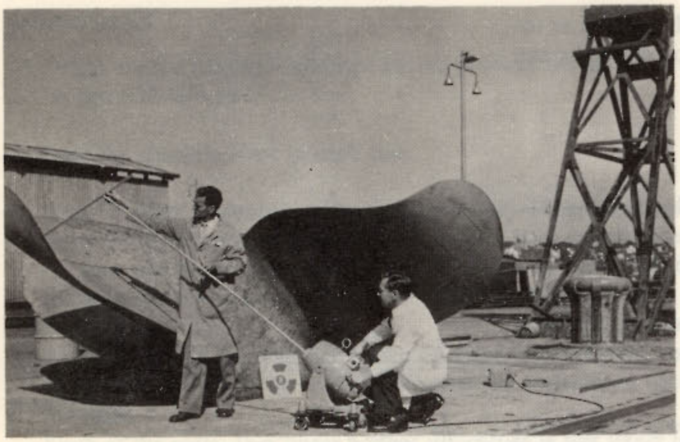


Fig 7.3 *A bent ship's propeller is radiographed to check for internal fractures.*

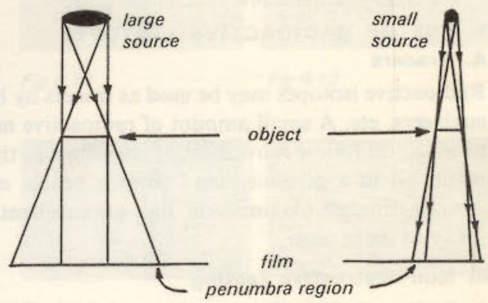


Fig 7.4

4. Using gamma or X-ray sources, "pictures" may be taken of castings, etc., to ensure that they have no flaws. Flaws show out on the subsequent radiograph. This method is used as an alternative to ultra-sonic techniques. To obtain a clear picture the following precautions should be taken:

(a) The source should be as small as possible. This consequently means that it must have a high specific activity (curie/mg). This has two effects. It reduces self-absorption, i.e. absorption of the radiation released by atoms within the source mass by outer atoms. It also reduces the penumbra effect (fig 7.4) and enables a clearer image to be obtained.

(b) The distance between the source and object should be as great as possible, particularly when the object to be radiographed has appreciable thickness (fig 7.5(a) and (b)).

This leads to a more even intensity throughout the thickness of the object under test.

(c) The film should be as close as possible to the object (in contact if possible). This improves the sharpness of the image obtained (fig 7.6).

(d) The radiation should be as close as possible to normal incidence on the object.

(e) As far as possible, the plane of maximum interest in the object should be parallel to the film. This is often difficult to achieve and is largely dictated by the shape of the object.

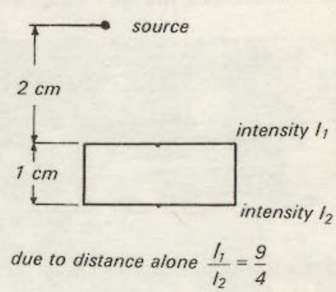


Fig 7.5(a)

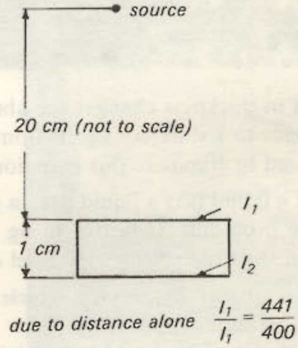
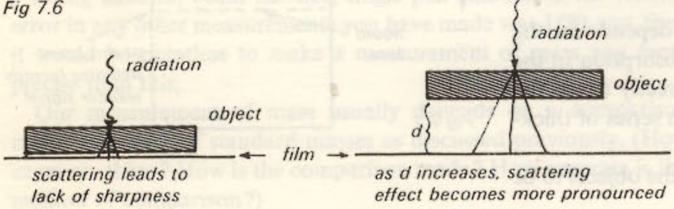


Fig 7.5(b)

Fig 7.6



2. LIGHT INTERFERENCE

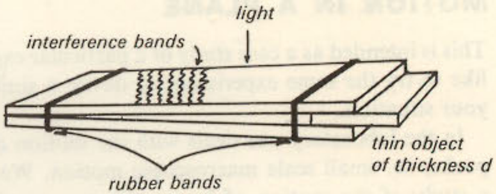


Fig 7.7

If an 'air wedge' is made using two microscope slides bound together with rubber bands with a thin object between them at one end, then the thickness of the thin object can be found in terms of the wavelength of the incident light, see fig 7.7.
This is usually the yellow light of a sodium vapour lamp.
By counting the number of dark bands seen and finding their average spacing, the thickness d can be found from

$$d = \frac{l\lambda}{2W} \quad \text{where} \begin{cases} W = \text{band spacing} \\ \lambda = \text{wave length of light} \\ l = \text{length of microscope slide} \end{cases}$$

This is an interference method for measuring lengths. Do not expect the bands to be exactly parallel (why not?). Remember that you are measuring in terms of wavelengths of light—something of the order of 6×10^{-7} metre.

3. NUCLEAR SCATTERING EXPERIMENTS

Read an account of Rutherford's work on the nucleus. Imagine that you were able to fire rubber balls (charged or uncharged) at an invisible object or array of objects and that your only source of information about the target was obtained from detecting what happened to the balls after they emerged from the target region.

If you fired enough projectiles and fired them at various speeds, then you could gradually build up a picture of the target—the order of magnitude of its size, whether it was charged or not, how "solid" it was, what could you knock out of it, etc.

Nuclear scattering experiments are devised in much the same way. This is hardly a measurement made on an undisturbed system though—in many cases the destruction of the original target results from the bombardment.

The targets are usually nuclei and the projectiles very energetic nucleons. Are electrons used? Discuss this.

4. MULTIFLASH METHODS

This method is discussed in detail in other sections of the course, so only brief mention will be made of it here. It is a useful method of investigating motion in two dimensions, and a large amount of information can be obtained from such photographs.

What limitations does this method have? Where do errors occur?

5. MOVIE FILM RECORDS

Slow-motion and high-speed films can often yield useful information that lends itself to assessment or measurement. Where do errors occur?

There are many experiments that can be carried out using relatively inexpensive movie equipment. The following is a description of such an experiment.

MOTION IN A PLANE

This is intended as a case study of a particular experiment. You may like to try the same experiment or devise a similar method to suit your situation.

In the laboratory one deals with the motion of carts and dry ice pucks, i.e. small scale macroscopic motion. We decided to try for a study of the motion of larger objects—people and basketballs. This also led to a more involved investigation of the errors involved.

A grid of 8 metres square was marked out using chalk in the school quadrangle and a large pendulum—period adjusted to 2 seconds—was set up on one side of this grid. Two basketballs were propelled across the grid in the first experiment and the action was photographed using an 8 mm camera situated on the roof of the school building near the quadrangle (fig 7.8).

After processing the film was run through a strip film projector, one frame at a time. It is a simple matter to make up a cardboard holder for an ordinary slide projector to enable this to be done. The camera speed had been set at 36 frames per second. Hence every 9 frames represented a time of 0.25 seconds. This proved an adequate time interval for the experiment. Some frames are shown in fig 7.9(a).

The positions of the two basketballs were marked at intervals of 0.25 secs by marking in their positions every 9 frames as the film was pulled through the slide viewer. At the same time the grid was marked in on the screen (a piece of stiff white cardboard). Fig 7.9(b) shows the result.

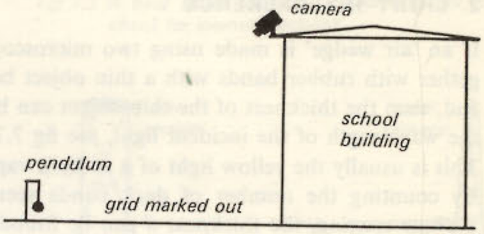
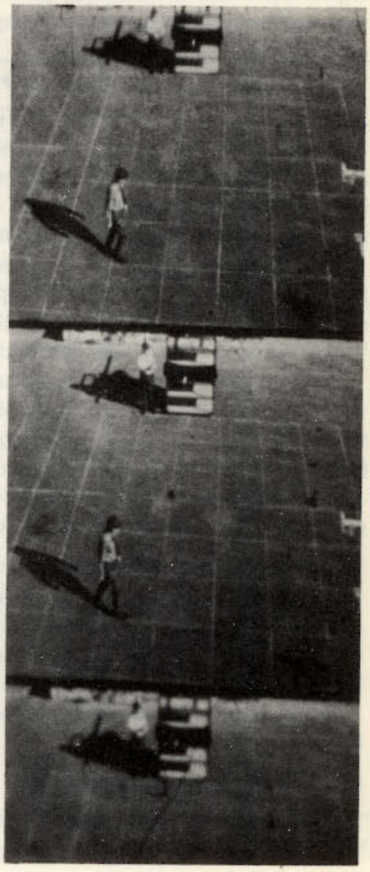


Fig 7.8

Fig 7.9(a)



time interval 0.25 seconds

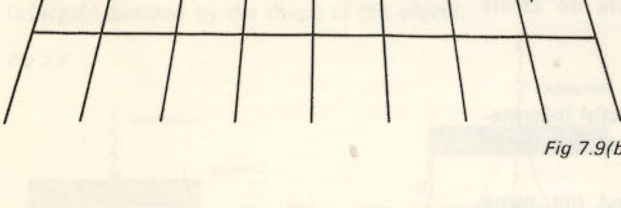


Fig 7.9(b)

QUESTIONS

1. It would appear that neither object has travelled in a straight line. Suggest at least two possible reasons for this.
2. The camera has recorded a perspective view and the square grid markings no longer appear square, i.e. we have a non-linear scale. Before reading on try to suggest a method for reading from this non-linear scale so that a graph of the movement of each of the objects can be drawn on a linear scale.
3. What was the *purpose of the pendulum*?

The photographic record was then transformed into a record of the event on a set of cartesian axes and shown in fig 7.10.

This was achieved by using an elastic scale. A length equal to the smallest division on the perspective view (fig 7.9(b)) was marked on a piece of white elastic. This was then divided into ten divisions, and by stretching the elastic each division could be correspondingly magnified. This enabled the positions of each object to be located as cartesian co-ordinates for each time interval.

For reference points in further discussion the points *A*, *B* and *C* were marked in (note that the frame of reference has been transposed or shifted—the base line in fig 7.9(b) is not the *X*-axis in fig 7.10).

Fig 7.10 represents a position table for the two basketballs and intervals of 0.25 secs.

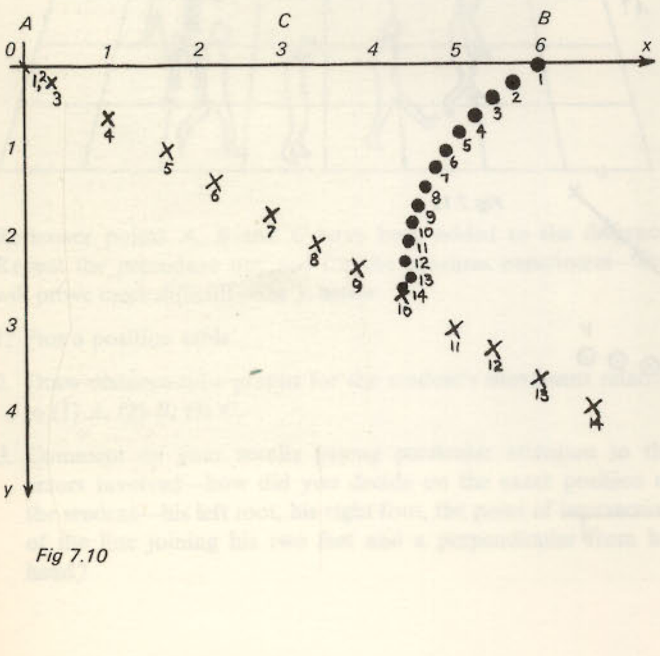


Fig 7.10

The following graphs were obtained for displacement from A and displacement from C respectively (figs 7.11 and 7.12).

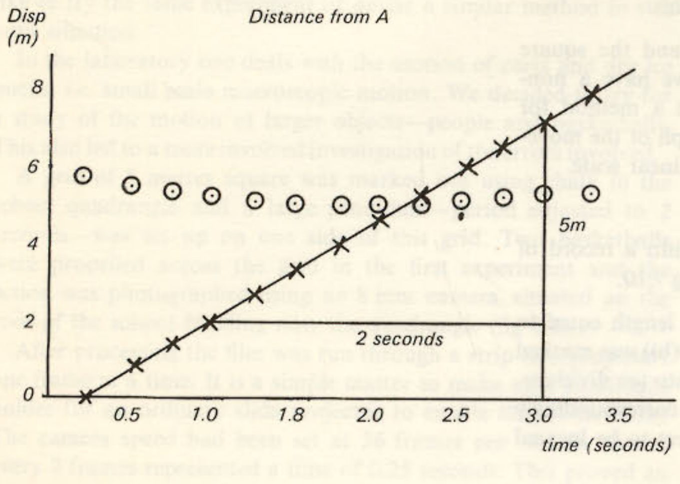


Fig 7.11

QUESTIONS—

1. How would you describe the motion of the object marked: (a) X (b) ⊙
   Suggest a reason for your answer in each case.
2. When were the objects equal distances from the point marked A in the position table?
3. Approximately when was the ⊙ object closest to A?

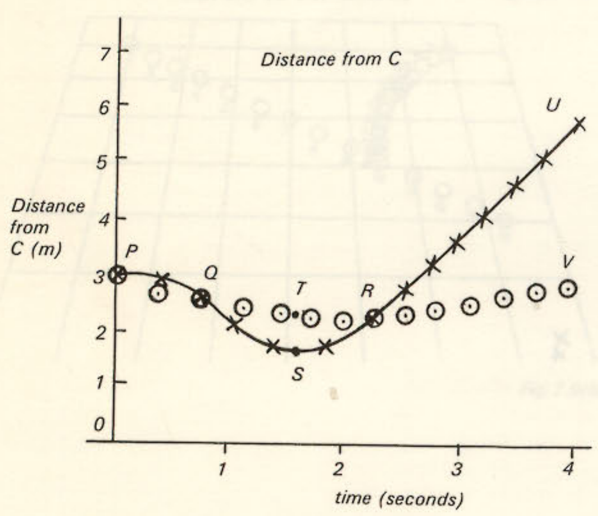


Fig 7.12

QUESTIONS—

1. What does the point *S* represent?
2. What do the points *Q* and *R* represent?
3. What do the slopes of the graphs in the regions *QS* and *QT* tell us about the motion of the objects relative to *C*?
4. What do the slopes of the graphs in the regions *RU* and *RV* tell us of the motion of the objects relative to *C*?
5. Try drawing a graph of the distances of each object from the point *B* for time intervals similar to those used in figs 7.11 and 7.12. Compare it with fig 7.11. Comment on your result.

The experiment was repeated using a student as the moving object instead of a basketball. The positions of the student were marked in on a grid as before, this time using every eighteenth movie film frame to give a time interval of 0.5 second. The result is shown in fig 7.13.

Time = 0.5 seconds

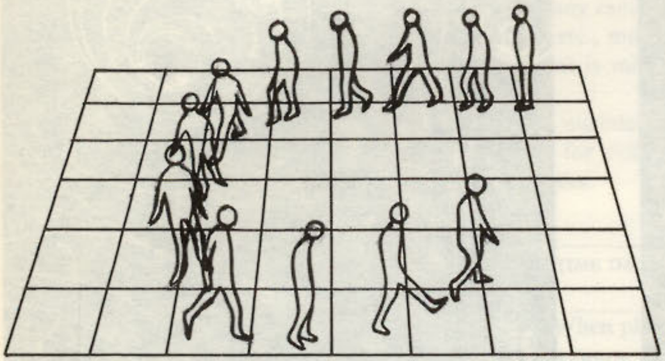


Fig 7.13

Reference points *A*, *B* and *C* have been added to the diagram. Repeat the procedure outlined for the previous experiment—this will prove more difficult—see 3. below.

1. Plot a position table.
2. Draw distance-time graphs for the student's movement relative to (1) *A*, (2) *B*, (3) *C*.
3. Comment on your results paying particular attention to the errors involved—how did you decide on the exact position of the student—his left root, his right foot, the point of intersection of the line joining his two feet and a perpendicular from his head?

6. PHOTOGRAPHIC EMULSIONS

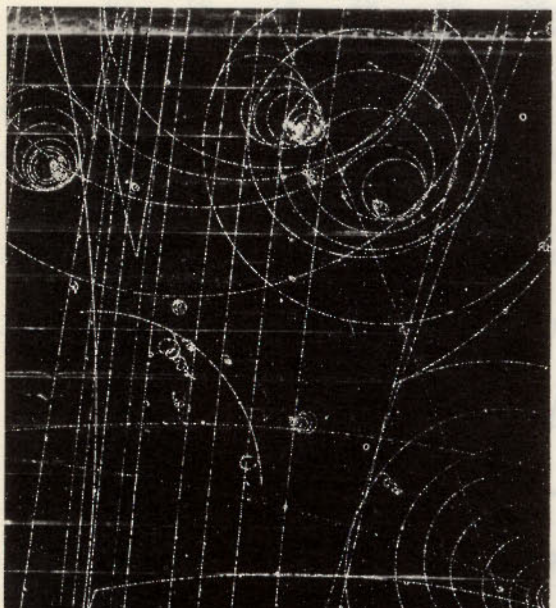
These can be used in high energy particle detection and analysis. Using special emulsions, primary particles and subsequent 'showers' of secondary particles can be identified and their energies measured.

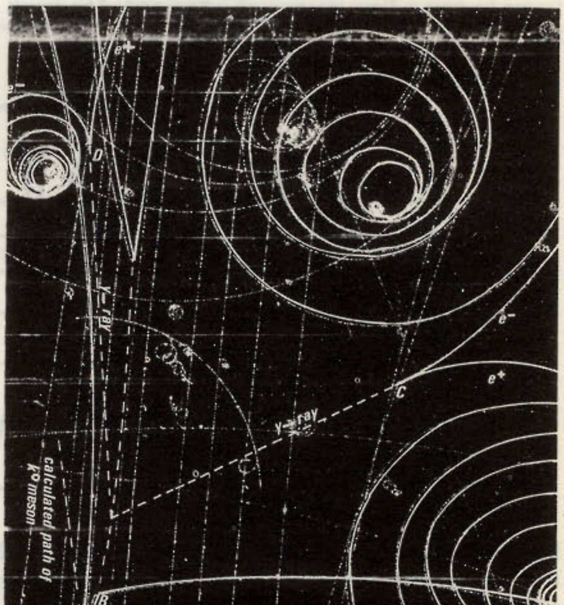
What do you think might limit the accuracy of any such measurement?

To help analyse the enormous amount of data, computers can be programmed to scan photographs, e.g. fig 7.14, checking them for unusual or sought after events. This takes some of the tedium out of human hands and also reduces errors (how?). How do you think it may increase the possibility of some other sort of error?

Find out how computers are used in this way as aids to measurement or even as measuring devices, checking events against some standard that is kept in the computer memory.

Fig. 7.14 Most of the lines in this bubble chamber photograph (except the straight ones) were produced by a collision which took place between a negative K-meson and a hydrogen nucleus (proton). The annotated version shows what happened. The main scientific interest in the photo is the track between A and B, the first recorded example of a negatively-charged Omega meson, whose properties had previously been predicted theoretically. Despite the prodigious velocities of the particles, the Omega meson travelled only about a centimetre before decaying. Its life could be calculated at approximately one ten billionth (10^{-10}) second.
Note also that the paths of neutral particles (which leave no track in the bubble chamber) have been dotted in on the annotated version. They can be calculated by the analysis of the tracks of their decay products (e.g. the electron-position pairs produced by the decay of the gamma rays at C and D) or by application of the laws of conservation of mass and energy (e.g. the neutral K-meson produced at A).





7. CARBON DATING

Radioactivity measurement can be used to date ancient organic material. To illustrate this we shall briefly discuss the Carbon-14 method of dating.

Carbon-14 is an unstable isotope of normal Carbon-12. Plants 'fix' atmospheric carbon dioxide while they are alive during the process of photosynthesis. The carbon dioxide from the air is formed into carbohydrates in the plant. Some of the carbon in carbon dioxide is the unstable Carbon-14 formed in the atmosphere by cosmic ray bombardment. Once the plant (or whatever has eaten the plant and hence transferred the Carbon-14 to its body) dies, the amount of Carbon-14 present in its structure cannot be increased.

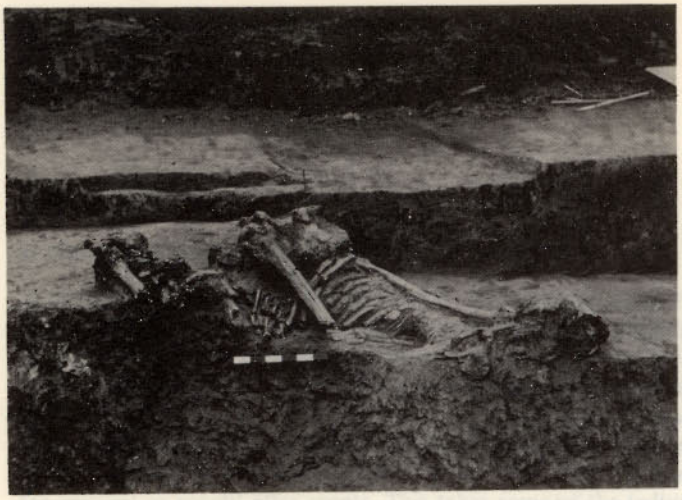
The unstable isotope decays at a known rate. It has a half-life of about 5568 years, i.e. a mass of Carbon-14 decays at such a rate that its mass is halved every 5568 years.

By comparing the amount of Carbon-14 present in a fossil with the amount of Carbon-14 originally in the plant, we can calculate the age of the fossil—or at least the time of death. This method assumes that the amount of Carbon-14 present in the atmosphere has remained constant. This is not strictly so. Why?

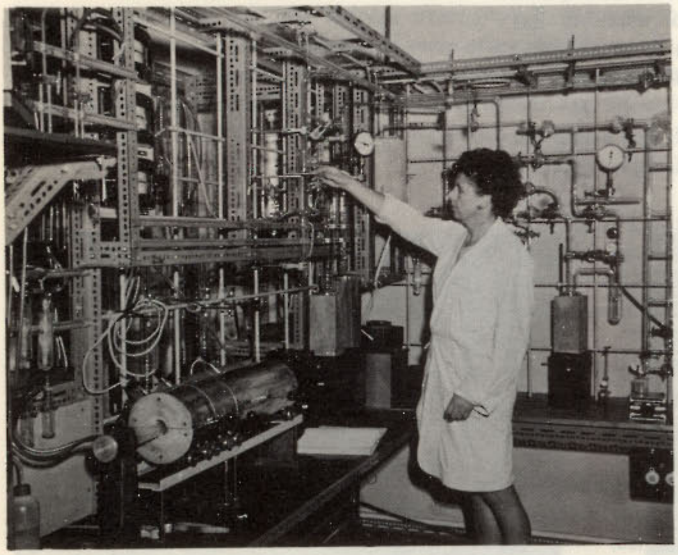
It is important not to oversimplify the method. In any radiation count, the *background count* due to cosmic radiation, etc., must be allowed for, and as the amount of Carbon-14 present is minute, great care must be taken in the count of its activity.

Other naturally occurring isotopes or radioactive nuclides are also used. Each has its advantages and each is used for different types of fossils. The table, fig 7.15, gives some examples.

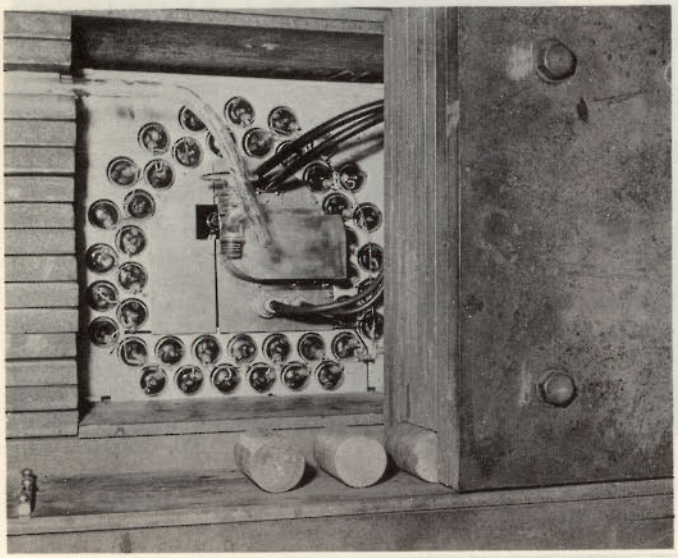
METHOD	MATERIAL	TIME DATED	USEFUL TIME SPAN (YEARS)
Carbon-14	Wood, peat, charcoal	When plant died	1000-50 000
	Bone, shell	Slightly before animal died	2000-35 000
Potassium-argon	Mica, some whole rocks	When rock last cooled to about 300°C	100 000 and up
	Hornblende Sanidine	When rock last cooled to about 500°C	10 000 000 and up
Rubidium-strontium	Mica	When rock last cooled to about 300°C	5 000 000 and up
	Potash feldspar	When rock last cooled to about 500°C	50 000 000 and up
	Whole rock	Time of separation of the rock as a closed unit	100 000 000 and up
Uranium-lead	Zircon	When crystals formed	200 000 000 and up
Uranium-238 fission	Many	When rock last cooled	100—1000 000 000 (Depending on material)



a



b



c

Fig 7.16 These human bones were discovered at Keilor, in Victoria, in 1965. The specimen was originally called 'Green Gulley Man', though it now seems that they are the bones of two people, a man and a woman.

Carbon-14 tests on the charcoal round the bones gave a date of (8155±130) years B.P., while the bones themselves were dated (6460±190) years B.P. (The B.P. stands for before present, 'present' being taken as 1950 A.D. Hence all ages refer to the number of years before 1950.)

The discrepancy between the two datings could be explained by assuming that the bones were buried in older material, or that the bone carbon has suffered greater contamination from newer carbon seeping through the soil than the carbon in the charcoal.

Carbon exists in bone in two forms—in the collogen and as calcium carbonate. In this sample, the collogen, which is less likely to be contaminated, provided the age of 6460 years, whereas the carbonate gave a figure of (1781±115) years—a pretty clear indication of substantial contamination.

An important point to bear in mind is that, although these dates are comparable with one another, the exact half-life of carbon has not been agreed upon. It is taken as (5568±30) years, and this figure is used by all laboratories to ensure consistency. However, it may well be 3% higher than this, in which case the quoted figures of age will have to be increased. Note, however, that the order of antiquity will be unchanged.

Photo (b) shows the preparation of the sample. It is placed in the cylindrical furnace (bottom left) where it is burned in pure oxygen. The carbon dioxide produced is then purified and passed through a vacuum system to the counting chamber (photo (c)). It is moved round the system by placing a thermos flask of liquid nitrogen around one of the collection bulbs in the system. Three of these flasks are visible in the photo, one just behind the operator.

Photo (c) shows the counting chamber itself. The sample is introduced via the glass tube into a counter hidden in the middle. The heavy iron door is then shut (what is the purpose of the cylinders in the bottom of the photo?).

The shielding keeps out almost all background radiation, but high-energy cosmic mesons can penetrate it. This is the reason for the ring of geiger tubes round the counter. Any simultaneous count from the central counter and one of the outside ones must be due to background radiation. These are known as 'coincident counts', and are deducted from the total count to give the actual count due to the sample.

APPENDIX 1: THE S.I. UNITS

One has only to look at a number of older physics text books to realise that some standard system of units is needed in the world. The names of many units in use today are not systematic and in many cases the same physical quantity is described in terms of many different, inadequately-designed units. This necessitates conversion of units and makes reading of texts using different systems of units a difficult task.

To ease communication a set of internationally recognised units with one unit for each quantity regarded as a separate physical quantity has been devised. It is called the International System of Units (S.I. for short).

The basic S.I. units are given in the following table. This is not yet complete. The basic physical quantity for amount of substance is proposed to be called the *mole* and to have the symbol 'mol'. This has not yet been adopted.

All physical quantities should now be expressed in terms of these units, Units that are not S.I. units or coherent with S.I. units should be used as little as possible. It is hoped that progressive discouragement of non S.I. units will lead to them being eventually abandoned.

The symbols should be used exactly as indicated. Note that the unit of thermodynamic temperature, K, does not have a degree sign as in °C. For example, the triple point of water (0°C) is 273.16K.

Some of the derived S.I. units are given in the following table with their approved S.I. names and definitions expressed in terms of the basic S.I. units.

BASIC PHYSICAL QUANTITY	NAME OF BASIC S.I. UNIT	SYMBOL
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Plane angle	radian	rad
Solid angle	steradian	sr

PREFIXES FOR METRIC (S.I.) UNITS

MULTIPLYING FACTOR	PREFIX	SYMBOL
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

PHYSICAL QUANTITY	NAME OF S.I. UNIT	S.I. SYMBOL	DEFINITION
Frequency	hertz	Hz	s^{-1}
Energy	joule	J	$kg\ m^2\ s^{-2}$
Force	newton	N	$kg\ m\ s^{-2} = J\ m^{-1}$
Power	watt	W	$kg\ m^2\ s^{-3} = J\ s^{-1}$
Electric charge	coulomb	C	A s
Electric potential difference	volt	V	$kg\ m^2\ s^{-3}\ A^{-1} = J\ A^{-1}\ s^{-1}$
Electric Resistance	ohm	Ω	$kg\ m^2\ s^{-3}\ A^{-2} = VA^{-1}$
Electric Capacitance	farad	F	$A^2\ s^4\ kg^{-1}\ m^{-2} = A\ s\ V^{-1}$
Magnetic flux	weber	Wb	$kg\ m^2\ s^{-2}\ A^{-1} = V\ s$
Inductance	henry	H	$kg\ m^2\ s^{-2}\ A^{-2} = V\ A^{-1}\ s$
Magnetic flux density	tesla	T	$kg\ s^{-2}\ A^{-1} = V\ s\ m^{-2}$
Luminous flux	lumen	lm	cd sr
Illumination	lux	lx	$cd\ sr\ m^{-2} = lm\ m^{-2}$

APPENDIX 2: PHYSICAL CONSTANTS AND CONVERSION FACTORS

LENGTH

	cm	metre	kilometre	inch	foot	mile
1 centimetre	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 metre	10^2	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 kilometre	10^5	10^3	1	3.937×10^4	3281	0.6214
1 inch	2.540	2.54×10^{-2}	2.54×10^{-5}	1	8.333	1.578×10^{-5}
1 foot	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mile	1.609×10^5	1609	1.609	6.336×10^4	5280	1

1 Angstrom unit $= 10^{-10}$ metre
1 micron $= 10^{-6}$ metre
1 nautical mile $= 1852$ metre
1 light year $= 9.4600\times10^{15}$ m
1 parsec $= 3.084\times10^{16}$ m

MASS

	g	kg	slug	A.M.U.
1 gram	1	10^{-3}	6.852×10^{-5}	6.024×10^{23}
1 kilogram	10^3	1	6.852×10^{-2}	6.024×10^{26}
1 slug	1.459×10^4	14.59	1	8.789×10^{27}
1 atomic Mass Unit	1.660×10^{-24}	1.660×10^{-27}	1.137×10^{-28}	1

A pound is not a mass unit although it is often used as such. However, a kilogram is a mass such that it "weighs" 2.205 lbs at the surface of the earth (approximately). This can be used as a conversion factor for terrestrial measurements.

ENERGY

	erg	joule	calorie	electron volt
1 erg	1	10^{-7}	2.389×10^{-8}	6.242×10^{11}
1 joule	10^7	1	0.2389	6.242×10^{18}
1 calorie	4.186×10^7	4.186	1	2.613×10^{19}
1 electron volt	1.602×10^{-12}	1.602×10^{-19}	3.827×10^{-20}	1

1 foot pound $= 1.356$ joule
1 kilowatt hour $= 3.6\times10^6$ joule
1 horsepower hour $= 2.685\times10^6$ joule
1 British Thermal Unit $= 1055$ joule

NAME	VALUE
speed of light	3.000×10^{8} metre second $^{-1}$
elementary charge	1.602×10^{-19} coulomb
Avogadro constant	6.023×10^{23} mole $^{-1}$
electron rest mass	9.109×10^{-31} kilogram
proton rest mass	1.673×10^{-27} kilogram
neutron rest mass	1.675×10^{-27} kilogram
Planck constant	6.626×10^{-34} joule second
gravitational constant	6.670×10^{-11} newton metre^{2} kilogram $^{-2}$
standard acceleration of free fall	9.807 metre second $^{-2}$
normal atmospheric pressure	1.013×10^{5} newton metre $^{-2}$
equatorial radius of earth	6.378×10^{6} metre
polar radius of earth	6.357×10^{6} metre
mass of earth	5.983×10^{24} kilogram
solar constant	1.340×10^{3} watt metre $^{-2}$
Earth—sun distance (mean)	1.49×10^{11} metre
Earth—moon distance (mean)	3.84×10^{8} metre
diameter of sun	1.39×10^{9} metre
mass of sun	1.99×10^{30} kilogram

APPENDIX 3: PROGRAMMED EXERCISE—ERRORS

This section is intended to help you understand how to use your knowledge of errors in numerical calculations. It should also indicate to you how to set out such answers step by step in a clear, logical way.

Cover up the right hand column with a piece of cardboard and gradually slide it down the page as you progress through the steps on the left hand side of the page.

1. The length of a rectangular bar is given as $(10.05 \pm .05)$ metre. The cross sectional area is given in M.K.S. units as $(0.000\,40 \pm .000\,002)$	. . . . m^2
2. The absolute error in the length is . . . . metre.	. . . . 0.05
3. If we divide the absolute error in the length by the measured value of the length we obtain the . . . . error.	. . . . relative
4. The numerical value of this error can be found from 0.05/10.05, or, to state it in a better way . . . . We usually round off such errors to the nearest 10, 50 or 100 as the case may be.	. . . . 1/200
5. The relative error in the area is given by . . . . and when simplified this becomes	. . . . 0.000 002/0.0004 . . . . 1/200
6. Therefore we can say that these two errors have the same . . . . of . . . . It is desirable that quantities should have relative errors of approximately the same order of magnitude when they are to be used in calculations.	. . . . order . . . . magnitude
7. The volume of the bar can be found by multiplying the measured value of cross sectional area by the measured value of the length, i.e. the volume $= 10.05 \times 0.000\,4$ . . . . This can be simplified to a value of 0.004 02 m^3 or 4.02×10^{-3} m^3.	. . . . m^3
8. To find the possible error in this value we must . . . the . . . . . errors in the length and cross-sectional area.	. . . . add . . . . relative
9. Hence the relative error in the volume is	. . . . $\frac{1}{200} + \frac{1}{200} = \frac{1}{100}$
10. To change this into an absolute error, it must be multiplied by	. . . . the volume (4.02×10^{-3}) m^3
11. Hence the absolute error in the volume is $4.02 \times 10^{-3} \times \frac{1}{100} = 4.02 \times 10^{-5}$	. . . . m^3
12. The volume may now be stated as $(0.004\,02 \pm .000\,04)$ m^3.	

13. Now that you have been through a problem of this type once try the following problem which has been set out in a more condensed manner. If the length of a sheet of paper is (5.05 ± 0.05) cm and the width is $(2.00 \pm .02)$ cm find the area.

Absolute error in length	= 0.05	. . . . cm
Relative error in length	=	. . . . 1/100
Absolute error in width	=	. . . . 0.02 cm
Relative error in width	= . . .	. . . . 1/100
Area = length × width	=	. . . . 10.1 cm^2
Relative error in area	=	. . . . 1/50
Absolute error in area		
= Relative error × area	=	. . . . 0.2 cm^2
Hence area may be stated as		(10.1 ± 0.2) cm^2

APPENDIX 4: ERRORS IN COMPUTING

Some of the errors that can be made when working with computers are obvious. Errors in programming logic and incorrect data supplied to the computer will both lead to incorrect results, but there are other sources of error. The following is reproduced from *Fortran IV Programming and Computing*, by J. T. Golden.

"Consider the set of equations

$$1.00x+1.00y = 1$$
$$1.00x+1.01y = 2$$

The solution is $x = -99$ and $y = 100$.

A 2% error in one of the original coefficients due to measurement errors, say, changes the equations to

$$1.00x+1.00y = 1$$
$$1.00x+0.99y = 2$$

The solution is now $x = 101$ and $y = -100$".

From this it can be seen that a relatively small change of 2% in only one of the measured values has resulted in a change of about 200 in the result for one of the calculated values. This is clearly undesirable whether the equations are solved by computer or manually.

"Even if the coefficients are exact, round off errors alone can cause wide variances in the solution. Suppose the correct equations are

$$1.00x+1.000\,000\,00y = 0$$
$$1.00x+0.999\,999\,99y = 1$$

The solution is $x = 10^8$ and $y = -10^8$.

If round-off changes the original equations to

$$1.00x+1.000\,000\,00y = 0$$
$$1.00x+1.000\,000\,01y = 1$$

the solution now becomes

$$x = -10^8 \text{ and } y = 10^8.$$

Equations such as these whose solution is very sensitive to changes in the values of coefficients are termed ill-conditioned".

The example taken was a rather simple one, but with a larger set of equations involving more unknowns the results could fluctuate even more. One of the uses of computers is to solve large systems of equations in large numbers of unknowns. (How would you like a set of 100 equations with 100 unknowns to solve, even if you were sure the co-efficients were satisfactory?) The data from these calculations may then be used as input data for further calculation again involving errors.

The previous example illustrates how wide fluctuation may result from manipulation of measured quantities.

Rounding errors will always occur. For example a subset of the Fortran language used with the IBM 1620 computer only accepts and prints data to eight significant figures. All values must then be rounded to this degree of accuracy using this particular computer language.

Using the same computer one may write the instruction to calculate X squared in the following two ways:—

(a) $XSQ = X**2$

(b) $XSQ = X*X$

If the value of X read in was 2.000 000 0—

Method (b) would probably result in the answer for X squared being 4.000 000 0.

Method (a) involved different internal processes in the computer during which rounding will occur and the answer could be 3.999 999 9.

If this value were then used in a later part of the programme, more rounding errors could occur, and although the answer might be printed with 8 figures, they could hardly all be relied upon sufficiently to call them "significant".

Taking another result, if you were asked to supply the answer to the problem $3 \times \frac{1}{3} = ?$ You would answer immediately "one" (I hope!)

A computer must approximate the value of one over three (or one third), because it has no way of storing fractions as such. If the computer uses binary arithmetic (a number system with a base of 2 instead of 10 as our normal system has), then it has neither terminating decimal fraction representation nor terminating binary representation.

In an 8 digit machine $\frac{1}{3}$ is represented as

0.333 333 33.

Three times this is 0.999 999 99.

Hence the computer would not achieve a match between $3 \times \frac{1}{3}$ and 1.

We have taken simple examples; the problem becomes much greater when calculations may be performed that involve thousands of manipulations on a set of data. Where greater precision is required you may see the term *double precision* used in the programme. The meaning is obvious, the arithmetic processes in the computer is a little more arduous and time-consuming, and remember that computer time means money—a lot of it. Computer time can be expensive if a lot of it is required on a modern machine. Hence programmes should be efficient and a balance between required accuracy and cost must be struck.

APPENDIX 5: QUESTIONS—GENERAL

1. (a) What is your height in metres? Angstrom units? Light years? Rods? Parsecs?
   (b) What is your mass in kilograms? Atomic mass units? Slugs?

2. The relationship between energy (E), mass (m) and the speed of light (c) is: $E = mc^2$
   Given that $c = 3 \times 10^8$ m s^{-1}, express your mass in joules. How many *electron volts* in this? How many *calories*?
   "If used completely, the mass of an average student could be converted into sufficient energy to supply the state of South Australia for about 100 years". Examine this statement critically—do you think it is true?

3. Man, as we know him, has existed on this earth for about one million years. Say the Universe has existed for 10^{10} years so far. If the age of the Universe is taken as a day, for how many seconds has man existed?

4. If an atom were the size of an average man—say two metre—how big would a man then be?

5. What conditions should a good clock satisfy?

6. "Some of the everyday 'simple' things we take for granted day after day are really most difficult to even begin to understand". Discuss this statement.

7. Two measurements are made with the same school ruler. One is stated as 3.0 cm, the other as 2.0 cm. Look at your school ruler.
   (a) What is the likely absolute error in each value?
   (b) What is the relative error in each?
   (c) If these two values are to be multiplied to give an area, what is the uncertainty in the area?

8. The speed of an object is stated as (50.0 ± 2.5) m s^{-1}. The distance involved in this estimation was (100.0 ± 1.0) metre. What is the absolute error in the time measurement? Suggest a likely time measuring instrument.

9. Use the uncertainty principle to find the uncertainty in position for the following cases:—
   (a) A bullet travelling at 1500 *ft. sec*$^{-1}$ with an uncertainty of 0.1 % in this value (careful of units!).
   (b) A proton travelling at 10^6 m s^{-1} with an uncertainty of 0.01 % in speed.

10. In an experiment using an accelerator to accelerate electrons, the energy of the electrons was calculated as 100 MeV. How many joules is this?
    Take the mass of an electron as 9×10^{-31} kg and use the formula—
    $$\text{Kinetic energy} = \frac{1}{2} mv^2$$
    to calculate the velocity of these electrons.
    Does your result seem strange? If so, do not simply say "It's wrong!" and go back to sleep. Think about it—suggest a refinement in the above procedure that will remove the "strangeness" (if in fact you find it so).

11. In Geneva the CERN organisation has built a huge sub-atomic particle accelerator 840 feet in diameter. This is to be used to boost electrons almost to the speed of light. How long would it take such an electron to make a circuit around this subatomic racetrack? (Can you really comprehend the magnitudes of the quantities you are dealing with?)

12. Which of the following quantities has (a) the largest absolute error, (b) the smallest relative error?
    (a) (5.05 ± 0.05) cm
    (b) (10.000 ± 0.0001) metre
    (c) $(0.05 \pm .005)$ kilometre
    (d) (50.0 ± 2.0) millimetre

13. A block of material is measured and its dimensions are found to be:
    $$l = 10.00 \pm 0.05 \text{ cm}$$
    $$b = 5.00 \pm 0.05 \text{ cm}$$
    $$w = 4.00 \pm 0.05 \text{ cm}$$
    The mass is found to be 10.05 ± 0.1 kg. Comment on the relative accuracies of each measurement. (You should calculate the relative errors in each quantity first.)
    Which measuring instruments could have been used to make the measurements?
    Calculate:
    (a) the area $l \times b$
    (b) the volume $l \times b \times w$
    (c) the density
    Give the relative error and the absolute error for each quantity calculated.

14. Two lengths are measured and one is found to be 20.0 ± 0.1 metre, the other 18.0 ± 0.1 metre. Calculate the sum and difference of the two quantities, taking account of the degree of uncertainty involved. Comment on a comparison of the relative errors in the two results.

    This has an important consequence in computer programming—find out what it is.

15. The following readings of a particular length were taken by a class, all readings in metres and summarised as follows:—

Length	4.7	4.8	4.9	5.0	5.1	5.2	5.3
Number of times length was obtained	1	5	16	25	13	4	2

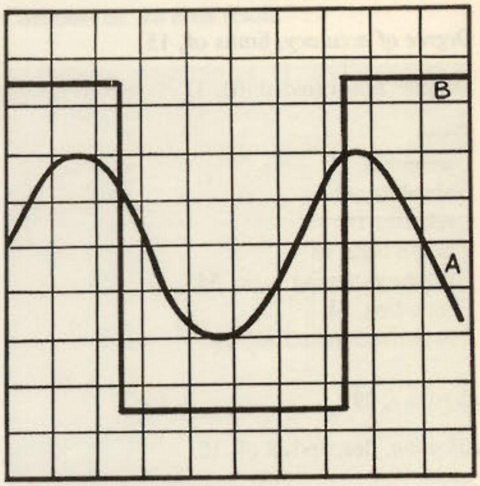
    Plot a graph of frequency of occurrence of measured length against length.

    (a) If these measurements were all made using the same object, what do you think the most likely value of the length was?

    (b) How could you modify an experiment such as this to obtain a more reliable value of length (without changing the object or the type of measuring instrument)?

16. Have a careful look at your watch or a clock of any kind. How accurately do you think you could state a time from your watch? i.e. state the time in the form $(A \pm a)$ units.

17. The volts/cm setting on a double-beam C.R.O. indicates 5 volts/cm and the time-base is set such that the time for a complete traverse of the graduated screen is one hundredth of a second for each beam. Examine the two traces below carefully. Scale is full size (1 cm = 1 cm) and the signals shown are periodic. Write down everything you can about these waveforms (amplitude, strength of signal, wavelength (?), frequency, shape, possible source, etc.).



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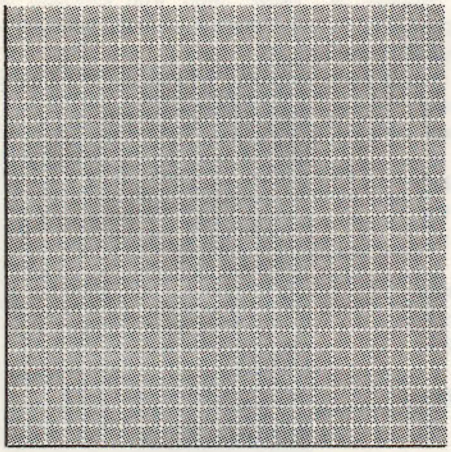
EXPERIMENT S7/1—ESTIMATION

This experiment involves the ability of an observer to look at two scale divisions and to estimate the value of some intermediate point. Computer cards are most useful in this experiment although anything similar will do.

If a set of such cards have a punching made in column 1 and another in column 50, with a third made somewhere in between, such that in a set of cards the third punching went from 2 to 49, the cards could then be shuffled to put them in random order. The cards could be lettered from A, B, C, etc. The observer would only be allowed to look at the back of the cards and would be told that the punching in the left hand side represented zero, that in the right hand side represented one. His problem is to estimate the value represented by the punching in the intermediate space and to record this estimation. A few seconds only should be allowed for each estimation. If the entire class performs the experiment the spread of results can be examined.

The actual value for each intermediate punching can be calculated by looking at the front of the card.

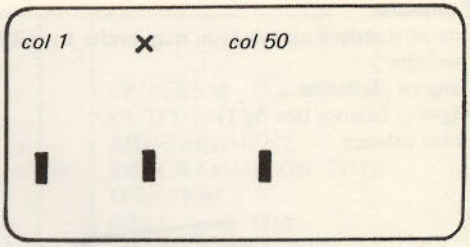
Plot the class results in the manner shown here.



No. of times estimate was made

Values estimated

Can you suggest any other method of plotting? What does this experiment tell you about your ability to interpolate when reading a scale?

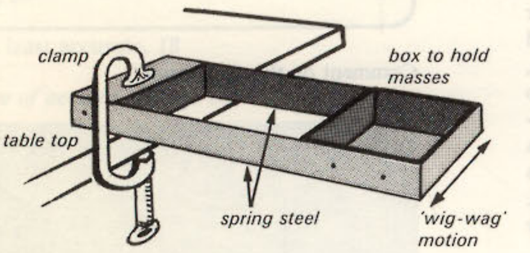


Comment on the class result.

EXPERIMENT S7/2—A WIG-WAG BALANCE

Equipment

box of standard masses (you may prefer to call them a box of "weights")
lump of plasticene
wig-wag balance (see fig 1)
beam balance



Take the mass of plasticene and place it in the left hand pan of the beam balance. Now add standard masses to the right hand pan until a balance is achieved. Record the values of the standard masses used. (Did you remember to see if the balance was in fact in equilibrium before you commenced the experiment?) Say this was x kg.

Now clamp the wig-wag balance firmly to the bench so that it can oscillate in a horizontal plane. Fix the plasticene in the box of the balance, pull it to one side, release it and count the number of complete oscillations made in one minute.

Remove the plasticene and place x kg of the standard masses in the box. Pull the box to one side to the same degree as before, release it and count the number of complete oscillations made in one minute.

How do these two mass comparison measurements differ in their method?

Which would be the most suitable form of mass comparison in an orbiting satellite? Why?

What can you conclude from these experiments?

EXPERIMENT S7/3: A RELATIVITY COMPUTATION

(This experiment can only be done if you have access to the use of a computer).

It can be shown using the theory of relativity that the length of a second as measured by an observer depends on the relative velocity of the observer with respect to the clock used for the measurement. Before carrying on with the experiment you should consult a reference book for more information on this effect.

The relation of time for clock in motion (*tm*), time for stationary clock (*ts*), velocity of moving clock (*v*) and the speed of light (*c*) is:—

$$tm = \sqrt{\frac{ts}{1 - v^2/c^2}}$$

In the FORTRAN programme which follows C represents the speed of light, TIMS represents the length of a time interval for a clock stationary with respect to an observer, TIMM represents the time interval when the clock is moving with velocity VELO with respect to the observer.

The programme indicates how time "slows" as velocity increases. You may be able to write a more efficient programme than this (I hope you can!). The number of the line on the left hand side is not included in the programme—it is there to aid in the description of the programme. Also, in an actual programme format statements should continue in the one line.

Number of Line			
1			PROGRAM TI (INPUT, OUTPUT)
2	C		MEASUREMENT
3	C	A	PROGRAMME ON TIME DILATION
4			READ 1, C, TIMS
5		1	FORMAT (E8.3, F4.1)
6			PRINT 20
7			PRINT 21
8		20	FORMAT (1H1, 13 HTIME DILATION, //)
9		21	FORMAT (5H TIME, 9X, 8HVELOCITY, //)
10			DO 9 I = 1,10
11			READ 2, VELO
12		2	FORMAT (E8.3)
13			TIMM = VELO 2./C 2
14			TIMM = 1.0—TIMM
15			IF (TIMM) 10, 10, 11
16		11	TIMM = SQRT(TIMM)
17			TIMM = TIMS/TIMM
18			PRINT 3, TIMM, VELO
19		3	FORMAT (1H, F8.3, E15.3)
20			GO TO 9
21		10	PRINT 13
22		13	FORMAT (9 H INFINITE)
23		9	CONTINUE
24			END

The information required is read in in lines 4 and 11. The format statements indicate how the information is to be read in. In the above programme you will have to supply all of the values of VELO (10 are required). If statement 2 were placed before statement 10, then only an initial value of velocity need be read in as data. Succeeding values could be generated by the statement

VELO = VELO+1.0E+7

after statement 19. This would increment VELO by 1.0E+7 at a rate of 1.0×10^6 units each time. Choice of the increment will depend on the initial value of VELO you choose.

What would be the effect of the following statement in place of the one above?

VELO = VELO+(C—VELO)/2.

Would statement 22 ever be printed out in this case?

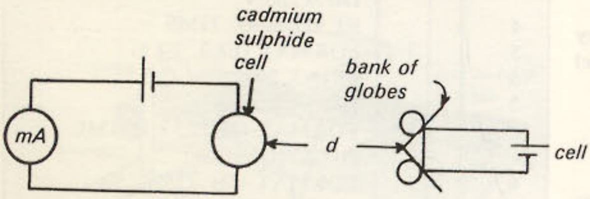
Lines 13 to 17 represent the coding of the formula in FORTRAN (this could be done more concisely) and the testing of the values to ensure that our clock has not exceeded the speed of light. Lines 18 to 23 tell the computer to print out the information that has been calculated.

As a first run through the programme prepare a set of eleven data cards. The first should have the values of C and TIMS on it: the second say a value equal to 0.1C; the third 0.2C; the fourth 0.3C and so on.

Think about the real accuracy of the results printed out by the computer. Draw a graph of length of time interval *vs* speed. What conclusions can you draw from your results?

EXPERIMENT S7/4—MILLIAMMETER LIGHT METER CONVERSION

Most electrical meters employ a galvanometer somewhere in their construction. It is our aim in this experiment to use a galvanometer to measure or compare light intensities, i.e. to turn it into a light meter. To do this the following equipment will be required: 10 torch globes, 2 dry cell batteries, milliammeter, cadmium sulphide cell, several lengths of wire and blackened sheet of cartridge paper.



Wire up the globes so that they may be disconnected from the dry cell one by one.

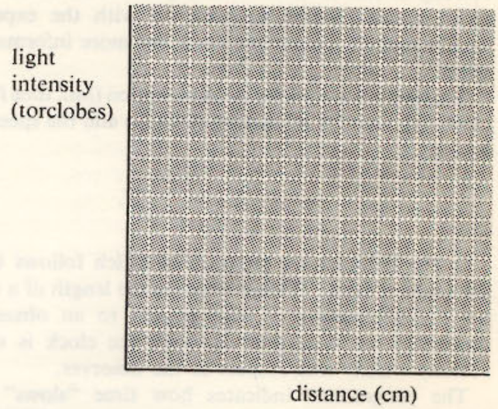
With the blackened paper in place, folded into a semi-cylindrical shape to cover the apparatus, select a convenient distance (e.g. 10 cm, 20 cm) from the cadmium sulphide cell such that the meter gives near full scale deflection with all ten globes on. The arrangement of apparatus is as shown above. It may be necessary to include a resistor in the CdS circuit to limit the current flowing to a safe value for the meter.

Use a felt-tip pen to mark the position of the pointer on the meter. This will represent a light intensity of 10 torclobes (a new 'unit' of light intensity) at a distance of *d* units. Switch off one of the globes and mark the new position of the pointer on the meter. Repeat until all globes are off. Does the pointer read zero on the milliammeter scale? Why?

You have now calibrated a milliammeter to read light intensities in torclobes. What would you have to do to change this into a more conventional unit?
(What in fact is the accepted unit of light intensity?) Can you think of any modifications that would make this into a more convenient and useful meter?

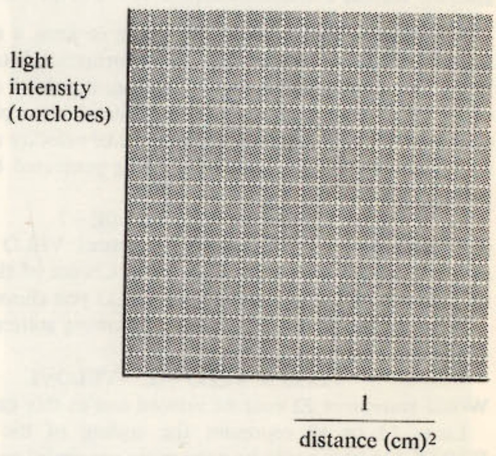
Having calibrated the light meter, let us now use it.

Record the readings on the meter with five torch globes on at distances of 4 cm, 6 cm, 8 cm, etc., from the cadmium sulphide cell. Plot this graph:



What type of curve do you obtain?

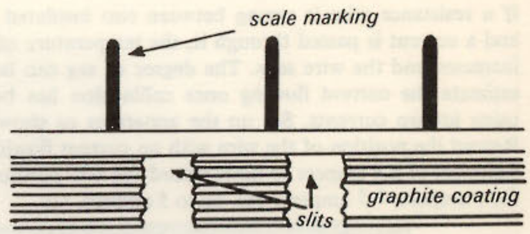
Now plot this graph:



What is the result?

Can you deduce the relationship between light intensity and distance from the source? How many other quantities obey this type of law?

EXPERIMENT S7/5—MEASUREMENT OF SMALL DISTANCES—INDIRECT METHODS



For this experiment you will need a microscope slide coated will colloidal graphite to make it opaque, a razor blade, transparent scale (e.g. a plastic set square), a school ruler and the use of a slide projector.

Using the razor blade, scratch two parallel lines across the surface of the graphite coated slide, as close together as possible.

The aim of the experiment is now to measure the width of each scratch and the separation of the two scratches as accurately as possible. There are many ways of doing this, but the following method is perhaps the simplest.

Place the slide in the projector and project a clear image of the scratches on a distant wall. Place a sheet of paper in the position of the image and rule in the image of the scratches on the slide. Now place the transparent scale in the projector and repeat the procedure. Use your school ruler to measure the magnification of the transparent scale—hence the width and separation of the scratches can be calculated knowing this magnification factor.

Repeat for various positions of the projector and of the slide in the projector.

Comment on the accuracy of your results.

RESULTS—

1. Can the accuracy of your results be improved indefinitely by enlarging the size of the image? (e.g. by moving the projector further away from the screen or wall). Explain.

2. Suggest another method to measure scratch width and separation.

3. What is the most difficult part about measuring the width of the scratch mark. (Can you in fact actually determine the boundaries clearly enough?) Estimate the percentage error in your results.

4. Using two razor blades bolted together, you could calculate the width of a razor blade using this method. How could this be done?

EXPERIMENT S7/6—A HOT WIRE AMMETER

If a resistance wire is strung between two insulated supports and a current is passed through it, the temperature of the wire increases and the wire sags. The degree of sag can be used to estimate the current flowing once calibration has been done using known currents. Set up the apparatus as shown above. Record the position of the wire with no current flowing. Allow a current of 0.5 ampere to flow, record the new position, repeat for 1 ampere, 1.5 ampere, etc. up to 5 ampere, say.

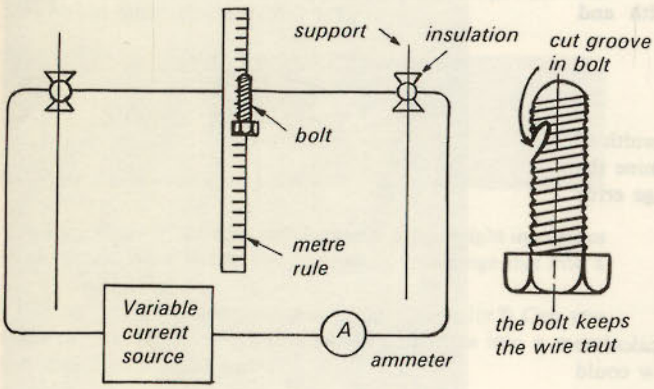
Current	
Reading	

Now reduce the current, retracing the values used before to check that the wire returns to its previous zero value. (If it has been stretched too tightly initially it will not return to zero—why?)

Provided that the apparatus is left in this condition it may now be used to measure electric current in ampere. What is a "linear scale?" Does this instrument have such a scale?

What are the limitations and possible sources of error of this instrument?

Do you know of any similar applications of the heating effect of electric current to measure the magnitude of the current (or vice-versa)?



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